

FLUX: FunctionaL Updates for XML (extended report)

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Abstract

XML database query languages have been studied extensively, but XML database updates have received relatively little attention, and pose many challenges to language design. We are developing an XML update language called FLUX, which stands for FunctionaL Updates for XML, drawing upon ideas from functional programming languages. In prior work, we have introduced a core language for FLUX with a clear operational semantics and a sound, decidable static type system based on regular expression types.

Our initial proposal had several limitations. First, it lacked support for recursive types or update procedures. Second, although a high-level source language can easily be translated to the core language, it is difficult to propagate meaningful type errors from the core language back to the source. Third, certain updates are well-formed yet contain *path errors*, or “dead” subexpressions which never do any useful work. It would be useful to detect path errors, since they often represent errors or optimization opportunities.

In this paper, we address all three limitations. Specifically, we present an improved, sound type system that handles recursion. We also formalize a source update language and give a translation to the core language that *preserves* and *reflects* typability. We also develop a *path-error analysis* (a form of dead-code analysis) for updates.

Categories and Subject Descriptors D.3.1 [Programming Languages]: Formal Definitions and Theory; H.2.3 [Database management systems]: Languages—data manipulation languages

General Terms Languages

Keywords XML, update languages, type systems, static analysis

1. Introduction

XQuery is a World Wide Web Consortium (W3C) standard, typed, purely functional language intended as a high-level interface for querying XML databases. It is meant to play a role for XML databases analogous to that played by SQL for relational databases. The operational semantics and type system of XQuery 1.0 has been formalized (Draper et al. 2007), and the W3C recently endorsed the formal semantics as a *recommendation*, the most mature phase for W3C standards.

Almost all useful databases change over time. The SQL standard describes a *data manipulation language* (DML), or, more briefly, *update language*, which facilitates the most common patterns of changes to relational databases: insertion, deletion, and in-place modification of rows, as well as addition or deletion of columns or tables. Despite the effectful nature of these operations, their semantics is still relatively clear and high-level. SQL updates are relatively inexpressive, but they are considered sufficient for most situations, as witnessed by the fact that in many SQL databases, data can *only* be updated using SQL updates in transactions. Moreover, the presence of SQL updates does no damage

to the purely functional nature of SQL queries: updates are syntactically distinct from queries, and the language design and transactional mechanisms ensure that aliasing difficulties cannot arise, even when an update changes the structure of the database (for example, if a column is added or removed from a table).

The XQuery standard lacks update language features analogous to SQL’s DML. While XML querying has been the subject of a massive research and development effort, high-level XML update languages have received comparatively little attention. Many programming languages for transforming *immutable* XML trees have been studied, including XML stylesheets (XSLT (Clark 1999)), and XML programming languages such as XDuce, CDuce, Xtatic, or OCamlDuce (Hosoya and Pierce 2003; Benzaken et al. 2003; Gapeyev et al. 2006; Frisch 2006). However, these languages are not well-suited to specifying updates. Updates typically change a small part of the document and leave most of the data fixed. To simulate this behavior by transforming immutable XML values one must explicitly describe how the transformation preserves unchanged parts of the the input. Such transformations are typically executed by building a new version of the document and then replacing the old one. This is inefficient when most of the data is unchanged. Worse, XML databases may employ auxiliary data structures (such as indices) or invariants (such as validity or key constraints) which need to be maintained when an update occurs, and updating a database by deleting its old version and loading a new version forces indices and constraints to be re-evaluated for the whole database, rather than incrementally.

Instead, therefore, several languages specifically tailored for updating XML data *in-place* have been proposed. While the earliest proposal, called XUpdate (Laux and Martin 2000), was relatively simple and has been widely adopted, it lacks first-class conditional and looping constructs. These features have been incorporated into more recent proposals (Tatarinov et al. 2001; Sur et al. 2004; Ghelli et al. 2006; Chamberlin et al. 2006; Ghelli et al. 2007b). The W3C is also developing a standard XQuery Update Facility (Chamberlin et al. 2008).

Although they have some advantages over XUpdate, we argue that these approaches all have significant drawbacks, because they unwisely combine imperative update operations with XQuery’s purely-functional query expressions. We shall focus our critique on XQuery!, since it is representative of several other proposals, including the W3C’s XQuery Update Facility.

A defining principle of XQuery! is that update operations should be “fully compositional”, which Ghelli et al. (2006) take to mean that an update operation should be allowed anywhere in an XQuery expression. Thus, the atomic update operations such as insertion, deletion, replacement, renaming, etc. may all appear within XQuery’s query expressions. Node-identifiers can be used as mutable references. To avoid nondeterminism, XQuery! fixes a left-to-right evaluation order and employs a two-phase semantics that first collects updates into a *pending update list* by evaluating an expression without altering the data, and then performs all of

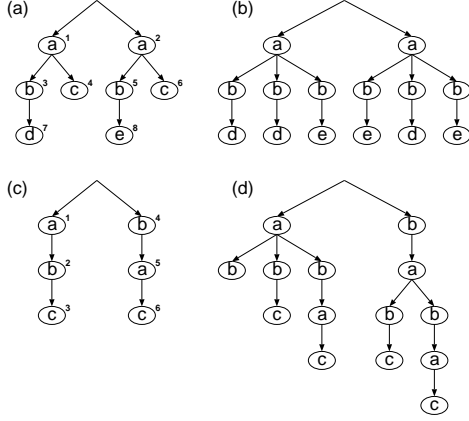


Figure 1. XQuery! examples

the updates at once. An additional operator called `snap` provides programmer control over when to apply pending updates.

XQuery! seems to sacrifice most of the good properties of XQuery. Most equational laws for XQuery expressions are invalid for XQuery!, and the semantics is highly sensitive to arbitrary choices. For example, consider the following XQuery! update.

```
for $x in $doc//a,
  $y in $doc//b
return (insert $y into $x, delete $x//c)
```

Its behavior on two trees is shown in Figure 1. In the first example, consider input tree (a) with regular structure. Running the above update on this tree yields the update sequence:

```
insert(1,<b><d/><b>); delete(4);
insert(1,<b><e/><b>); delete(4);
insert(2,<b><d/><b>); delete(6);
insert(2,<b><e/><b>); delete(6);
```

The numbers refer to the node identifiers shown in Figure 1(a) as superscripts. When these updates are performed, the result is output (b). Note that each subtree labeled `a` in the output contains three `b`-subtrees, one corresponding to the original `b` and one for each occurrence of `b` in the tree. In the second example, tree (c) is transformed to (d) via updates

```
insert(1,<b><c/><b>); delete(3);
insert(1,<b><a><c/><a/><b/>); delete(3);
insert(5,<b><c/><b>); delete(6);
insert(5,<b><a><c/><a/><b/>); delete(6);
```

Observe that both occurrences of `a` have as subtrees both occurrences of `b` in the input. This is because the snapshot semantics of XQuery! first collects the updates to be performed, then performs them in sequence. Inserts always copy data from the original version of the document, whereas deletes mark nodes in the new version of the document for deletion. This is why some occurrences of `c` remain below occurrences of `a`. Although this is not an update that a user would typically write, an implementation, type system, or static analysis must handle all of the expressions in the language, not just the well-behaved ones.

Furthermore, the XQuery! approach seems quite difficult to statically typecheck. There are several reasons for this. First, as the examples in Figure 1 show, the structure of the result can depend on the data in ways difficult to predict using types. Second, XQuery! also permits side-effects to be made visible before the end of an update, using a “snapshot” operator called `snap`. The `snap`

operator forces all of the delayed side-effects of an expression to be performed. This means that the values of variables may change during an update, so it would be necessary to update the types of variables to typecheck such updates. Since variables may alias parts of the document, this requires a nontrivial alias analysis.

We argue that the combination of features considered in XQuery! and similar proposals are unnecessarily complex for the problem of updating XML databases. While high expressiveness is certainly a reasonable design goal, we believe that for XML database updates, expressiveness must be balanced against other concerns, such as semantic transparency and static typechecking. We believe that it is worthwhile to consider an alternative approach that sacrifices expressiveness for semantic clarity and the ability to typecheck and analyze *typical* updates *easily*.

In previous work (Cheney 2007), we introduced a core *Functional Update language for XML*, called FLUX.¹ FLUX is *functional* in the same sense that imperative programming in Haskell using monads² is functional. Side-effects may be present, but they are encapsulated using syntactic and type constraints, so that queries remain purely functional. FLUX provides sufficient expressive power to handle most common examples of XML database-style updates (e.g. all of the relational use cases in the XQuery Update Facility Requirements (Chamberlin and Robie 2005)), while avoiding complications resulting from the interaction of unrestricted iteration and unconstrained side-effects.

FLUX admits a relatively simple, one-pass operational semantics, so side-effects can be performed eagerly; it can also be typechecked using regular expression types, extending previous work on typechecking XQuery (Hosoya et al. 2005; Colazzo et al. 2006; Draper et al. 2007). The decidability of typechecking for the core language (with recursive types and functions) was later established by Cheney (2008) along with a related result for an XQuery core language. However, our preliminary proposal (Cheney 2007) had several limitations. This paper presents an improved design. In particular, our contributions relative to prior work are:

- We extend the core language with recursive types and update procedures and provide a sound type system.
- We adapt the idea of *path-error analysis*, introduced by Colazzo et al. (2006) for a core XML query language, to the setting of updates, and design a correct static analysis for conservatively under-approximating update path-errors.
- We formalize a high-level FLUX source language and show how to translate to core FLUX. We also present a source-level type system and prove that a source update is well-formed *if and only if* its translation is well-formed.

The structure of the rest of this paper is as follows. Section 2 briefly recapitulates the FLUX source language introduced in (Cheney 2007) with a few examples. Section 3 formalizes the core language and its operational semantics; its type system is presented and proved sound in Section 4. Section 5 presents the translation from the high-level FLUX language to Core FLUX and shows how to typecheck high-level updates. Section 6 presents the path-error analysis. We provide a more detailed comparison with related approaches in Section 7; Section 8 presents extensions and future work; and Section 9 concludes.

Certain definitions and proofs have been placed in appendices.

¹originally LUX, for “Lightweight Updates for XML”.

²Technically, FLUX’s approach to typechecking updates is closer to *arrows* (Hughes 2000); however, we will not investigate this relationship in detail here.

```

Stmt ::= Upd [WHERE Expr]
        | IF Expr THEN Stmt
        | Stmt; Stmt'
        | LET Var := Expr IN Stmt
        | {Stmt}

Upd ::= INSERT (BEFORE|AFTER) Path VALUE Expr
        | INSERT AS (LAST|FIRST) INTO Path VALUE Expr
        | DELETE [FROM] Path
        | RENAME Path TO Lab
        | REPLACE [IN] Path WITH Expr
        | UPDATE Path BY Stmt

Path ::= . | Lab | node() | text()
        | Path/Path | Var AS Path | Path[Expr]

```

Figure 2. Concrete syntax of FLUX updates.

2. Overview and examples

2.1 Syntax

As with many similar languages, particularly SQL, XQuery (Draper et al. 2007), and CPL+ (Liefke and Davidson 1999), we will introduce a high-level, readable source language syntax which we will translate to a much simpler core language. We will later formalize the operational semantics and type system for the core language. In what follows, we assume familiarity with XQuery and XPath syntax, and with XDuce-style regular expression types.

The high-level syntax of FLUX updates is shown in Figure 2. XQuery variables *Var* are typically written $\$x$, $\$y$, etc. We omit the syntactic class *Expr* consisting of ordinary XQuery expressions, respectively. Statements *Stmt* include conditionals, let-binding, sequential composition, and update statements *Upd*, which may be guarded by a WHERE-clause. We use braces to parenthesize statements $\{Stmt\}$. Updates *Upd* come in two flavors, *singular* and *plural*. Singular updates expect a single tree and are executed once for each selected tree; plural updates operate on arbitrary sequences and are executed on the children of each selected tree. Singular insertions (INSERT BEFORE/AFTER) insert a value before or after each node selected by the path expression, while plural insertions (INSERT AS FIRST/LAST INTO) insert a value at the beginning or end of the child-list of each selected node. Similarly, singular deletes (DELETE) delete individual nodes selected by the given *Path*, whereas plural deletes (DELETE FROM) delete the child-list of each selected node. Singular replacement REPLACE WITH replaces a subtree, while plural replacement REPLACE IN replaces the content of a path with new content. The renaming operation RENAME TO is always singular; it renames a subtree’s label. The UPDATE *Path* BY *Stmt* operation is singular; it applies *Stmt* to each tree matching *Path*. Update procedure declarations are not shown but can be added easily to the source language.

The *path expressions* *Path* in FLUX are based on the XPath expressions that are allowed in XQuery. Paths include the empty path \cdot , sequential composition *Path/Path*, the XPath child axis tests (`text()`, `\Lab` and `node()`), filters (*Path*[*Expr*]), and variable binding steps (*Var AS Path*). The “as” path expression $\$x$ AS *Path* (not present in XPath) binds the subtree matching *Path* to $\$x$ in each iteration of a path update. We often write $*$ instead of `node()`. We only describe the syntax of paths used to perform *updates*; arbitrary XPath or XQuery expressions may be used in subqueries *Expr*.

Both the FLUX source language described here and the core language introduced later are case-insensitive with respect to keywords (like XQuery); however, we use uppercase for the source language and lowercase for the core language to prevent confusion.

2.2 Execution model, informally

In general, an update is evaluated as follows: The path expression is evaluated, yielding a *focus selection*, or a set of parts of the updatable store on which the update *focuses*. The WHERE-clause, if present, is evaluated with respect to the variables bound in the path and if the result is `true` then the corresponding basic update operation (insert, delete, etc.) is applied to each element of this set in turn. Order of evaluation is unspecified and the semantics is consistent with parallel evaluation of iterations.

Unlike most other proposals, in FLUX, arbitrary XPath or XQuery expressions cannot be used to select foci. If this were allowed, it would be easy to construct examples for which the result of an update depends on the order in which the focus selection is processed. For example, suppose the document is of the form `<a>`. If the following update were allowed:

```
UPDATE /* BY { DELETE a/b;    RENAME * TO c }
```

then the result would depend on the order in which the updates are applied. Two possible results are `<c/>` and `<c><c/></c>`. This nondeterministic behavior is difficult to typecheck. For this reason, we place severe restrictions on the path expressions that may be used to select foci.

We identify two key properties which help to ensure that updates are deterministic and can be typechecked. First, *an update can only modify data at or beneath its current focus*. We call this the *side-effect isolation property*. For example, navigating to the focused value’s parent and then modifying a sibling is not allowed. In addition, whenever we perform an iterative update traversing a number of nodes, we require that *the result of an iterative update is independent of the order in which the nodes are updated*. We call this the *traversal-order independence property*.

To ensure isolation of side effects and traversal-order independence, it is sufficient to restrict the XPath expressions that can be used to select foci. Specifically, only the child axis³ is allowed, and absolute paths starting with `/` cannot be used to backtrack to the root of the document in order to begin iterating over some other part. This ensures that only descendants of a given focused value can be selected as the new focus and that a selection contains no overlapping parts. Consequently, the side effects of an update are confined to the subtrees of its focus, and the result of an iteration is independent of the traversal order. This keeps the semantics deterministic and helps make typechecking feasible.

2.3 Examples

Suppose we start an XML database with no pre-loaded data; its type is `db[()]`. We want to create a small database listing books and authors. The following FLUX updates accomplish this:

```
U1 : INSERT AS LAST INTO db VALUE books[] ;
      INSERT AS LAST INTO db VALUE authors[]
```

After this update, the database has type

```
books[] , authors[]
```

Suppose we want to load some XML data into the database. Since XML text is included in XQuery’s expression language, we can just do the following:

```
U2 : INSERT INTO books VALUE
      <book><author>Charles Dickens</author>
      <title>A Tale of Two Cities</title>
      <year>1858</year></book>
      <book><author>Lewis Carroll</author>
```

³The attribute axis can also be handled easily, but the descendant, parent, and sibling axes seem nontrivial to handle.

```

        <title>Alice in Wonderland</title>
        <year>??</year></book>;
INSERT INTO authors VALUE
<author><name>Charles Dickens</name>
        <born>1812</born>
        <died>1870</died></author>
<author><name>Lewis Carroll</name>
        <born>1832</born>
        <died>1898</died></author>

```

This results in a database with type

```

books[ book[author[string],title[string],
        year[string]]* ],
authors[ author[name[string],born[string],
        died[string]]* ]

```

The data we initially inserted had some missing dates. We can fill these in as follows:

```

U3 : UPDATE $x AS books/book BY
      REPLACE IN year WITH "1859"
      WHERE $x/title/text() = "A Tale of Two Cities"
U4 : UPDATE $x AS books/book BY
      REPLACE IN year WITH "1865"
      WHERE $x/title/text() = "Alice in Wonderland"

```

Note that here, we use an XQuery expression `$x/name/text()` for the WHERE-clause. Both updates leave the structure of the database unchanged.

We can add an element to each book in books as follows:

```

U5 : INSERT AS LAST INTO books/book
      VALUE publisher["Grinch"]

```

After U5, the books database has type

```

books[ book[author[string],title[string],
        year[string],publisher[string]]* ]

```

Now perhaps we want to add a co-author; for example, perhaps Lewis Carroll collaborated on “Alice in Wonderland” with Charles Dickens. This is not as easy as adding the publisher field to the end because we need to select a particular node to insert before or after. In this case we happen to know that there is only one author, so we can insert after that; however, this would be incorrect if there were multiple authors, and we would have to do something else (such as inserting before the title).

```

U6 : UPDATE $x AS books/book BY
      INSERT AFTER author
      VALUE <author>Charles Dickens</author>
      WHERE $x/name/text() = "Alice in Wonderland"

```

Now the books part of the database has the type:

```

books[ book[author[string]*,title[string],
        year[string],publisher[string]]* ]

```

Now that some books have multiple authors, we might want to change the flat author lists to nested lists:

```

U7 : REPLACE $x AS books/book WITH
      <book><authors>{$x/author}</authors>
      {$x/title}{$x/year}{$x/publisher}</book>

```

This visits each book and changes its structure so that the authors are grouped into an authors element. The resulting books subtree has type:

```

books[ book[authors[author[string]* ],title[string],
        year[string],publisher[string]]* ]

```

Suppose we later decide that the publisher field is unnecessary after all. We can get rid of it using the following update:

```

U8 : DELETE books/book/publisher

```

The books subtree in the result has type

```

books[ book[authors[author[string]* ],
        title[string],year[string]]* ]

```

Now suppose Lewis Carroll retires and we wish to remove all of his books from the database.

```

U9 : DELETE $x AS books/book
      WHERE $x/authors/author/text() = "Lewis Carroll"

```

This update does not modify the type of the database. Finally, we can delete a top-level document as follows:

```

U10 : DELETE authors

```

2.4 Non-design goals

There are several things that other proposals for updating XML do that we make no attempt to do. We believe that these design choices are well-motivated for FLUX’s intended application area, database updates.

Node identity: The XQuery data model provides identifiers for all nodes. Many XML update proposals take node identities into account and can use them as to update parts of the tree “by reference”. In contrast, FLUX’s semantics is purely value-based. Although there are currently no examples involving node identity for XQuery database updates in the W3C’s requirements documents (Chamberlin and Robie 2005), node identity is important in other XML update settings such as the W3C’s Document Object Model (DOM). We believe it is possible to adapt FLUX to a data model with node identity as long as the identifiers are not used as mutable references.

Pattern matching: Many transformation/query languages (e.g. (Hosoya et al. 2005; Clark 1999)) and some update languages (e.g. (Liefke and Davidson 1999; Wang et al. 2003)) allow defining transformations by *pattern matching*, that is, matching tree patterns against the data. Pattern matching is very useful for XML transformations in Web programming (e.g. converting an XML document into HTML), but we believe it is not as important for typical XML database updates. We have not considered general pattern matching in FLUX, in order to keep the type system and operational semantics as simple as possible.

Side-effects in queries: Several motivating examples for XQuery! (Ghelli et al. 2006) and XQueryP (Chamberlin et al. 2006) depend on the ability to perform side-effects within queries. Examples include logging accesses to particular data or profiling or debugging an XQuery program. FLUX cannot be used for these applications. However, it is debatable whether adding side-effects to XQuery is the best way to support logging, profiling, or debugging for XQuery.

3. Core language formalization

The high-level update language introduced in the last section is convenient for users, but its operations are complex, overlapping, and difficult to typecheck. Just as for XQuery and many other languages, it is more convenient to define a core language with orthogonal operations whose semantics and typing rules are simple and transparent, and then translate the high-level language to the core language.⁴ We first review the XML data model, regular expression types, and the μ XQ core query language of Colazzo et al. (2006).

⁴Such core languages are also typically easier to optimize, though we do not consider optimization in this paper.

3.1 XML values and regular expression types

Following Colazzo et al. (2006), we distinguish between *tree values* $t \in Tree$, which include strings $w \in \Sigma^*$ (for some alphabet Σ), boolean values $\text{true}, \text{false} \in Bool$, and singleton trees $n[v]$ where $n \in Lab$ is a node label; and (*forest*) values $v \in Val = Tree^*$, which are sequences of tree values:

Tree values $t ::= n[v] \mid w \mid \text{true} \mid \text{false}$
 (Forest) values $v ::= () \mid t, v$

We overload the set membership symbol \in for trees and forests: that is, $t \in v$ means that t is a member of v considered as a list. Two forest values can be concatenated by concatenating them as lists; abusing notation, we identify trees t with singleton forests $t, ()$ and write v, v' for forest concatenation. We define a comprehension operation on forest values as follows:

$[f(x) \mid x \in ()] = ()$
 $[f(x) \mid x \in t, v] = f(t), [f(x) \mid x \in v]$

This operation takes a forest (t_1, \dots, t_n) and a function $f(x)$ from trees to forests and applies f to each tree t_i , concatenating the resulting forests in order. Comprehensions satisfy basic monad laws as well as some additional equations (see (Fernandez et al. 2001)). We use $=$ for (mathematical) equality of tree or forest values.

We consider a regular expression type system with structural subtyping, similar to those considered in several transformation and query languages for XML (Hosoya et al. 2005; Colazzo et al. 2006; Fernandez et al. 2001).

Atomic types $\alpha ::= \text{bool} \mid \text{string} \mid n[\tau]$
 Sequence types $\tau, \sigma ::= \alpha \mid () \mid \tau \mid \tau' \mid \tau, \tau' \mid \tau^* \mid X$

We call types of the form $\alpha \in Atom$ atomic types (or sometimes tree or singular types), and types $\tau, \sigma \in Type$ of all other forms *sequence types* (or sometimes forest or plural types). Sequence types are constructed using regular expression operations such as the empty sequence $()$, alternative choice $\tau \mid \tau'$, sequential composition τ, τ' and iteration (or Kleene star) τ^* . Type variables $X \in TyVar$ denoting recursively defined types are also allowed; these must be declared in signatures as discussed below.

A value of singular type must always be a sequence of length one (that is, a tree, string, or boolean); plural types may have values of any length. There exist plural types with only values of length one, but which are not syntactically singular (for example $\text{string} \mid \text{bool}$). As usual, the $+$ and $?$ quantifiers are definable as follows: $\tau^+ = \tau, \tau^*$ and $\tau^? = \tau \mid ()$.

We define *type definitions* and *signatures* as follows:

Type definitions $\tau_0 ::= \alpha \mid () \mid \tau_0 \mid \tau'_0 \mid \tau_0, \tau'_0 \mid \tau_0^*$
 Type signatures $E ::= \cdot \mid E, \text{type } X = \tau_0$

Type definitions τ_0 are types with no top-level variables (that is, every variable is enclosed in a $n[-]$ context). A signature E is well-formed if all type variables appearing in definitions are also declared in E . Given a well-formed signature E , we write $E(X)$ for the definition of X . A type τ denotes the set of values $[\tau]_E$, defined as follows.

$[\text{string}]_E = \Sigma^* \quad [n[\tau]]_E = \{n[v] \mid v \in [\tau]_E\}$
 $[\text{bool}]_E = Bool \quad [\tau \mid \tau']_E = [\tau]_E \cup [\tau']_E$
 $[()]_E = \{()\} \quad [X]_E = [E(X)]$
 $[\tau, \tau']_E = \{v, v' \mid v \in [\tau]_E, v' \in [\tau']_E\}$
 $[\tau^*]_E = \bigcup_{n=0}^{\infty} \{v_1, \dots, v_n \mid v_1, \dots, v_n \in [\tau]_E\}$

Formally, $[\tau]_E$ is defined by a straightforward least fixed point construction which we omit (see e.g. (Hosoya et al. 2005)). Henceforth, we treat E as fixed and define $[\tau] \triangleq [\tau]_E$. This se-

mantics validates standard identities such as associativity of $'$, $([\tau_1, \tau_2], \tau_3) = [\tau_1, (\tau_2, \tau_3)]$, unit laws $([\tau, ()] = [\tau] = [(), \tau])$, and idempotence of $'^*$ ($[(\tau^*)^*] = [\tau^*]$).

A type τ_1 is a *subtype* of τ_2 ($\tau_1 <: \tau_2$), by definition, if $[\tau_1] \subseteq [\tau_2]$. The use of regular expressions (including untagged unions) for XML typing poses a number of problems for subtyping and typechecking which have been resolved in previous work on XDuce (Hosoya et al. 2005). Our types are essentially the same as those used in XDuce, so subtyping reduces to XDuce subtyping; although this problem is EXPTIME-complete in general, the algorithm of Hosoya et al. (2005) is well-behaved in practice. Therefore, we shall not give explicit inference rules for checking or deciding subtyping, but treat it as a “black box”.

3.2 Core query language

Because FLUX uses queries for insertion, replacement, and conditionals, we need to introduce a query language and define its semantics before doing the same for FLUX. In our implementation, we use a variant of the μXQ core language introduced by Colazzo et al. (2006), which has the following syntax:

$e ::= () \mid e, e' \mid n[e] \mid w \mid x \mid \text{let } x = e \text{ in } e'$
 $\mid \text{true} \mid \text{false} \mid \text{if } c \text{ then } e \text{ else } e' \mid e \approx e'$
 $\mid \bar{x} \mid \bar{x}/\text{child} \mid e :: n \mid \text{for } \bar{x} \in e \text{ return } e'$

We follow the convention in (Colazzo et al. 2006) of using \bar{x} for variables introduced by `for`, which are always bound to tree values; ordinary variables x may be bound to any value.

An *environment* is a pair of functions $\gamma : (Var \rightarrow Val) \times (TVar \rightarrow Tree)$. Abusing notation, we write $\gamma(x)$ for $\pi_1(\gamma)(x)$ and $\gamma(\bar{x})$ for $\pi_2(\gamma)(\bar{x})$; similarly, $\gamma[x := v]$ and $\gamma[\bar{x} := t]$ denote the corresponding environment updating operations. The semantics of queries is defined via the large-step operational semantics judgment $\gamma \vdash e \Rightarrow v$, meaning “in environment γ , expression e evaluates to value v ”. The contributions of this paper do not require detailed understanding of the query language, so the rules are relegated to the appendix. We omit recursive queries but they can be added without difficulty.

3.3 Core update language

We now introduce the core FLUX update language, which includes statements $s \in Stmt$, tests $\phi \in Test$, and directions $d \in Dir$:

$s ::= \text{skip} \mid s; s' \mid \text{if } e \text{ then } s \text{ else } s' \mid \text{let } x = e \text{ in } s$
 $\mid \text{insert } e \mid \text{delete} \mid \text{rename } n$
 $\mid \text{snapshot } x \text{ in } s \mid \phi?s \mid d[s] \mid P(\bar{e})$
 $\phi ::= n \mid \text{node}() \mid \text{text}()$
 $d ::= \text{left} \mid \text{right} \mid \text{children} \mid \text{iter}$

Here, P denotes an *update procedure* name. Procedures are defined via declarations $P(\bar{x} : \bar{\tau}) : \tau_1 \Rightarrow \tau_2 \triangleq s$, meaning P takes parameters \bar{x} of types $\bar{\tau}$ and changes a database of type τ_1 to one of type τ_2 . We collect these declarations into a set Δ , which we take to be fixed throughout the rest of the paper. Procedures may be recursive.

Updates include standard constructs such as the no-op `skip`, sequential composition, conditionals, and `let`-binding. Recall that updates work by *focusing* on selected parts of the mutable store. The basic update operations include insertion `insert e`, which inserts a value provided the focus is the empty sequence; deletion `delete`, which deletes the focus (replacing it with the empty sequence); and `rename n`, which renames the current focused value (provided it is a singleton tree). The “snapshot” operation `snapshot x in s` binds x to the current focused value and then applies an update s , which may refer to x . There is no way to re-

$$\boxed{\gamma; v \vdash s \Rightarrow^u v'}$$

$$\begin{array}{c}
\frac{}{\gamma; v \vdash \text{skip} \Rightarrow^u v} \quad \frac{\gamma; v \vdash s \Rightarrow^u v_1 \quad \gamma; v_1 \vdash s' \Rightarrow^u v_2}{\gamma; v \vdash s; s' \Rightarrow^u v_2} \quad \frac{\gamma \vdash e \Rightarrow \text{true} \quad \gamma; v \vdash s_1 \Rightarrow^u v'}{\gamma; v \vdash \text{if } e \text{ then } s_1 \text{ else } s_2 \Rightarrow^u v'} \quad \frac{\gamma \vdash e \Rightarrow \text{false} \quad \gamma; v \vdash s_2 \Rightarrow^u v'}{\gamma; v \vdash \text{if } e \text{ then } s_1 \text{ else } s_2 \Rightarrow^u v'} \\
\frac{\gamma \vdash e \Rightarrow v \quad \gamma[x := v]; v_1 \vdash s \Rightarrow^u v_2}{\gamma; v_1 \vdash \text{let } x = e \text{ in } s \Rightarrow^u v_2} \quad \frac{\gamma \vdash e \Rightarrow v}{\gamma; () \vdash \text{insert } e \Rightarrow^u v} \quad \frac{}{\gamma; v \vdash \text{delete} \Rightarrow^u ()} \quad \frac{}{\gamma; n'[v] \vdash \text{rename } n \Rightarrow^u n[v]} \\
\frac{\gamma[x := v]; v \vdash s \Rightarrow^u v'}{\gamma; v \vdash \text{snapshot } x \text{ in } s \Rightarrow^u v'} \quad \frac{t \in \llbracket \phi \rrbracket \quad \gamma; t \vdash s \Rightarrow^u v}{\gamma; t \vdash \phi?s \Rightarrow^u v} \quad \frac{t \notin \llbracket \phi \rrbracket}{\gamma; t \vdash \phi?s \Rightarrow^u t} \quad \frac{}{\gamma; n[v] \vdash \text{children}[s] \Rightarrow^u n[v']} \\
\frac{\gamma; () \vdash s \Rightarrow^u v'}{\gamma; v \vdash \text{left}[s] \Rightarrow^u v', v} \quad \frac{\gamma; () \vdash s \Rightarrow^u v'}{\gamma; v \vdash \text{right}[s] \Rightarrow^u v, v'} \quad \frac{\gamma; t_1 \vdash s \Rightarrow^u v'_1 \quad \gamma; v_2 \vdash \text{iter}[s] \Rightarrow^u v'_2}{\gamma; t_1, v_2 \vdash \text{iter}[s] \Rightarrow^u v'_1, v'_2} \quad \frac{}{\gamma; () \vdash \text{iter}[s] \Rightarrow^u ()} \\
\frac{P(\vec{x} : \vec{\tau}) : \tau_1 \Rightarrow \tau_2 \triangleq s \in \Delta \quad \gamma \vdash e_1 \Rightarrow v_1 \quad \cdots \quad \gamma \vdash e_n \Rightarrow v_n \quad \gamma[x_1 := v_1, \dots, x_n := v_n]; v \vdash s \Rightarrow^u v'}{\gamma; v \vdash P(\vec{e}) \Rightarrow^u v'}
\end{array}$$

Figure 3. Operational semantics of updates.

fer to the focus of an update within a μ XQ query without using snapshot. Also, `snapshot` is *not* equivalent to XQuery!’s `snap` operator; `snapshot` binds x to an immutable value which can be used in s , whereas `snap` forces execution of pending updates in XQuery!.

Updates also include *tests* $\phi?s$ which allow us to examine the local structure of a tree value and perform an update if the structure matches. The node label test $n?s$ checks whether the focus is of the form $n[v]$, and if so executes s , otherwise is a no-op; the wildcard test `node()?` s only checks that the value is a singleton tree. Similarly, `text()?` s tests whether the focus is a string. The $?$ operator binds tightly; for example, $\phi?s; s' = (\phi?s); s'$.

Finally, updates include *navigation* operators that change the selected part of the tree and perform an update on the sub-selection. The `left` and `right` operators move to the left or right of a value. The `children` operator shifts focus to the children of a tree value. The `iter` operator shifts focus to all of the tree values in a forest.

We distinguish between *singular* (unary) updates which apply to tree values and *plural* (multi-ary) updates which apply to sequences. Tests $\phi?s$ are always singular. The `children` operator applies a plural update to the children of a single node; the `iter` operator applies a singular update to all of the elements of a sequence. Other updates can be either singular or plural in different situations.

Figure 3 shows the operational semantics of Core FLUX. We write $\gamma; v \vdash s \Rightarrow^u v'$ to indicate that given environment γ and focus v , statement s updates v to value v' . The rules for tests are defined in terms of the following semantic interpretation of tests:

$$\begin{aligned}
\llbracket \text{text}() \rrbracket &= \Sigma^* \\
\llbracket n \rrbracket &= \{n[v] \mid v \in \text{Val}\} \\
\llbracket \text{node}() \rrbracket &= \text{Tree}
\end{aligned}$$

Note that we define the semantics entirely in terms of forest and tree values, without needing to define an explicit store. This would not be the case if we considered full XQuery, which includes node identity comparison operations. However, we believe our semantics is compatible with allowing node-identity tests in queries.

Theorem 1 (Update determinism). *Let γ, v, s, v_1, v_2 be given such that $\gamma; v \vdash s \Rightarrow^u v_1$ and $\gamma; v \vdash s \Rightarrow^u v_2$. Then $v_1 = v_2$.*

Proof. Straightforward by induction on the structures of the two derivations. The interesting cases are those for conditionals, tests, and iteration, since they are the only statements that have more than one applicable rule. However, in each case, only matching pairs of rules are applicable. \square

4. Type system

As noted earlier, certain *singular* updates expect that the input value is a singleton (for example, `children`, $n?s$, etc.) while *plural* updates work for an arbitrary sequence of trees. Singular updates fail if applied to a sequence. Our type system should prevent such runtime failures. Moreover, as with all XML transformation languages, we often would like to ensure that when given an input tree of some type τ , an update is guaranteed to produce an output tree of some other type τ' . For example, updates made by non-privileged users are usually required to preserve the database schema.

We define a matching relation between tree types and tests: we say that $\alpha <: \phi$ if $\llbracket \alpha \rrbracket \subseteq \llbracket \phi \rrbracket$. This is decidable using the following rules:

$$\frac{}{\text{string} <: \text{text}()} \quad \frac{}{n[\tau] <: n} \quad \frac{}{\alpha <: \text{node}()}$$

We employ a type system for queries similar to that developed by Colazzo et al. (2006). We consider type environments Γ consisting of sets of bindings $x:\tau$ of variables to types and $\bar{x}:\alpha$ of tree variables to atomic types. (We never need to bind a tree variable to a sequence type). As usual, we assume that variables in type environments are distinct; this convention implicitly constrains all inference rules. We write $\llbracket \Gamma \rrbracket$ for the set of all environments γ such that $\gamma(x) \in \llbracket \Gamma(x) \rrbracket$ and $\gamma(\bar{x}) \in \llbracket \Gamma(\bar{x}) \rrbracket$ for all $x \in \text{dom}(\Gamma)$ and $\bar{x} \in \text{dom}(\Gamma)$ respectively.

The typing judgment for queries is $\Gamma \vdash e : \tau$, meaning *in type environment Γ , expression e has type τ* . The typing rules are essentially the same as those in (Colazzo et al. 2006).

The main typing judgment for updates is $\Gamma \vdash^a \{\tau\} s \{\tau'\}$, meaning *in type environment Γ , an a -ary update s maps values of type τ to type τ'* . Here, $a \in \{1, *\}$ is the arity of the update, and singular update judgments always have $\tau = \alpha$ atomic. In addition, we define auxiliary judgments $\Gamma \vdash_{\text{iter}} \{\tau\} s \{\tau'\}$ for typechecking iterations and $\vdash_{\text{Decl}} \Delta$ for typechecking declarations Δ . The rules for update well-formedness are shown in Figure 4.

4.1 Discussion

In many functional languages, and several XML update proposals, side-effecting operations are treated as expressions that return $()$. Thus, we could typecheck such updates as expressions of type $()$. This is straightforward provided the types of values reachable from the free variables in Γ do not change; for example, this is the case for ML-style references. However, if the side-effects do change the types of the values of variables, then Γ needs to be updated to take these changes into account. One possibility is to typecheck updates using a residuating judgment $\Gamma \vdash s : () \mid \Gamma'$; here, Γ' is the updated type environment reflecting the types of the variables after

$$\boxed{\Gamma \vdash^a \{\tau\} s \{\tau'\}}$$

$$\frac{\Gamma \vdash^a \{\tau\} s \{\tau'\} \quad \Gamma \vdash^a \{\tau'\} s' \{\tau''\}}{\Gamma \vdash^a \{\tau\} \text{skip } \{\tau\}}$$

$$\frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash^a \{\tau\} s \{\tau_1\} \quad \Gamma \vdash^a \{\tau\} s' \{\tau_2\}}{\Gamma \vdash^a \{\tau\} \text{if } e \text{ then } s \text{ else } s' \{\tau_1 | \tau_2\}}$$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash^* \{\emptyset\} \text{insert } e \{\tau\}} \quad \frac{\Gamma \vdash^a \{\tau\} \text{delete } \{\emptyset\}}{\Gamma \vdash^a \{\tau\} \text{delete } \{\emptyset\}}$$

$$\frac{\Gamma \vdash^a \{n[\tau]\} \text{rename } n \{n[\tau]\} \quad \Gamma \vdash^a \{\tau_1\} \text{let } x = e \text{ in } s \{\tau_2\}}{\Gamma \vdash^a \{n[\tau]\} \text{rename } n \{n[\tau]\}}$$

$$\frac{\Gamma, x : \tau \vdash^a \{\tau\} s \{\tau'\} \quad \alpha <: \phi \quad \Gamma \vdash^1 \{\alpha\} s \{\tau\}}{\Gamma \vdash^a \{\tau\} \text{snapshot } x \text{ in } s \{\tau'\}}$$

$$\frac{\alpha \not<: \phi \quad \Gamma \vdash^* \{\tau\} s \{\tau'\}}{\Gamma \vdash^1 \{\alpha\} \phi?s \{\alpha\}} \quad \frac{\Gamma \vdash^1 \{n[\tau]\} \text{children}[s] \{n[\tau']\}}{\Gamma \vdash^* \{\emptyset\} s \{\tau'\}}$$

$$\frac{\Gamma \vdash^a \{\tau\} \text{left}[s] \{\tau', \tau\} \quad \Gamma \vdash^a \{\tau\} \text{right}[s] \{\tau, \tau'\}}{\Gamma \vdash^a \{\tau\} \text{left}[s] \{\tau', \tau\}}$$

$$\frac{\Gamma \vdash^a \{\tau\} \text{iter}[s] \{\tau'\} \quad \Gamma \vdash^a \{\tau_1\} s \{\tau'_2\} \quad \tau'_2 <: \tau_2}{\Gamma \vdash^* \{\tau\} \text{iter}[s] \{\tau'\}}$$

$$\frac{P(\vec{x} : \vec{\tau}) : \sigma_1 \Rightarrow \sigma_2 \triangleq s \in \Delta \quad \sigma'_1 <: \sigma_1 \quad \Gamma \vdash e_1 : \tau'_1 \quad \tau'_1 <: \tau_1 \quad \dots \quad \Gamma \vdash e_n : \tau'_n \quad \tau'_n <: \tau_n}{\Gamma \vdash^a \{\sigma'_1\} P(\vec{e}) \{\sigma_2\}}$$

$$\boxed{\Gamma \vdash_{\text{iter}} \{\tau\} s \{\tau'\}}$$

$$\frac{\Gamma \vdash^1 \{\alpha\} s \{\tau\} \quad \Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau_2^*\}}{\Gamma \vdash_{\text{iter}} \{\emptyset\} s \{\emptyset\}}$$

$$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\}}{\Gamma \vdash_{\text{iter}} \{\tau_1, \tau_2\} s \{\tau'_1, \tau'_2\}}$$

$$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\}}{\Gamma \vdash_{\text{iter}} \{\tau_1 | \tau_2\} s \{\tau'_1 | \tau'_2\}}$$

$$\frac{\Gamma \vdash_{\text{iter}} \{E(X)\} s \{\tau\}}{\Gamma \vdash_{\text{iter}} \{X\} s \{\tau\}}$$

$$\boxed{\vdash_{\text{Decl}} \Delta}$$

$$\frac{\vdash_{\text{Decl}} \Delta \quad \vec{x} : \vec{\tau} \vdash^* \{\tau_1\} s \{\tau_2\}}{\vdash_{\text{Decl}} \emptyset \quad \vdash_{\text{Decl}} \Delta, P(\vec{x} : \vec{\tau}) : \tau_1 \Rightarrow \tau_2 \triangleq s}$$

Figure 4. Update, iteration, and declaration well-formedness.

update s . This approach quickly becomes complicated, especially if it is possible for variables to “alias”, or refer to overlapping parts of the data.

In FLUX, we take a completely different approach to typechecking updates. The judgment $\Gamma \vdash^a \{\tau\} s \{\tau'\}$ assigns an update much richer type information that describes the type of the updatable context before and after running s . The variables in Γ are immutable, so their types never need to be updated.

The most unusual rules are those involving the `iter`, `test`, and `children`, `left/right`, and `insert/rename/delete` operators. The following example illustrates how the rules work for these constructs. Consider the update:

`iter [a?children [iter [b?right [insert c]]]]`

Intuitively, this update inserts a c after every b under a top-level a . Now consider the input type $a[b\Box^*, c\Box, b\Box^*], d\Box$. Clearly, the output type *should* be $a[(b\Box, c\Box)^*, c\Box, (b\Box, c\Box)^*], d\Box$. To see why this is the case, first note that the following can be derived for any τ, τ', s :

$$\frac{\vdash^1 \{a[\tau]\} s \{a[\tau']\}}{\vdash^* \{a[\tau], d\Box\} \text{iter } [a?s] \{a[\tau'], d\Box\}}$$

Using the rule for `children`, we can see that it suffices to check that `iter [b?right [insert c]]` maps type $b\Box^*, c\Box, b\Box^*$ to $(b\Box, c\Box)^*, c\Box, (b\Box, c\Box)^*$. This is also an instance of a derivable rule

$$\frac{\vdash^1 \{b\Box\} s \{\tau\}}{\vdash^* \{b\Box^*, c\Box, b\Box^*\} \text{iter } [b?s] \{\tau^*, c\Box, \tau^*\}}$$

Hence, we now need to show only that `right [insert c]` maps type $b\Box$ to $b\Box, c\Box$, which is immediate:

$$\frac{\frac{\vdash^1 \{\emptyset\} : \emptyset}{\vdash^1 c(\emptyset) : c(\emptyset)}}{\vdash^* \{\emptyset\} \text{insert } c\Box \{c\Box\}}$$

$$\vdash^1 \{b\Box\} \text{right } [\text{insert } c\Box] \{b\Box, c\Box\}$$

4.2 Metatheory

We take for granted the following type soundness property for queries (this was proved for μXQ in Colazzo et al. (2006)).

Theorem 2 (Query soundness). *If $\Gamma \vdash e : \tau$ and $\gamma \in \llbracket \Gamma \rrbracket$ then $\gamma \vdash e \Rightarrow v$ implies $v \in \llbracket \tau \rrbracket$.*

The corresponding result also holds for updates, by a straightforward structural induction argument (presented in the appendix):

Theorem 3 (Update soundness). *Assume $\vdash_{\text{Decl}} \Delta$ holds.*

1. If $\Gamma \vdash^a \{\tau\} s \{\tau'\}$, $v \in \llbracket \tau \rrbracket$, and $\gamma \in \llbracket \Gamma \rrbracket$, then $\gamma; v \vdash s \Rightarrow^v v'$ implies $v' \in \llbracket \tau' \rrbracket$.
2. If $\Gamma \vdash_{\text{iter}} \{\tau\} s \{\tau'\}$, $v \in \llbracket \tau \rrbracket$, and $\gamma \in \llbracket \Gamma \rrbracket$, then $\gamma; v \vdash \text{iter}[s] \Rightarrow^v v'$ implies $v' \in \llbracket \tau' \rrbracket$.

Moreover, typechecking is decidable for both μXQ and FLUX in the presence of the subsumption rules (Cheney 2008).

5. Normalization

There is a significant gap between the high-level FLUX language we presented in Section 2 and the core language in the previous section. In this section, we formalize a translation from the source language presented in Section 2 to Core FLUX. In XQuery, this kind of translation is called *normalization*. We define three normalization functions called *path expression normalization* $[-]_{\text{Path}}(-)$, *update statement normalization* $[-]_{\text{Stmnt}}$, and *simple update normalization* $[-]_{\text{Upd}}$. These functions are defined in Figure 5.

Path expression normalization takes an extra parameter, which must be a core FLUX update; that is, $[p]_{\text{Path}}(s)$ normalizes a path p by expanding it to an expression which navigates to p and then does s . Compound statement normalization is straightforward. Simple updates are first normalized by translation to p . We omit the cases needed to handle WHERE-clauses; however, they can be handled by the existing translation if we consider e.g. `REPLACE p WITH e WHERE c` to be an abbreviation for `REPLACE $p[c]$ WITH e` , etc. In particular, note that the translation places both c and e into the scope of all variables declared in p .

Since the translation rules cover all cases and are orthogonal, it is straightforward to see that the normalization functions are total functions from the source language to Core FLUX.

5.1 Typechecking source updates

Normalization complicates type-error reporting, since we cannot always easily explain why the translation of an update fails to typecheck in source-level terms familiar to the user. We therefore also develop a type system for the source language that is *both sound and complete* with respect to core FLUX typechecking. This type system can therefore be used to report type errors to users in terms of the source language. We assume that query subexpressions e have already been normalized to μXQ according to the standard

$[u]_{Stmt} = [u]_{Upd}$	$[\text{INSERT BEFORE } p \text{ VALUE } e]_{Upd} = [p]_{Path}(\text{left}[\text{insert } e])$
$[\text{IF } e \text{ THEN } s]_{Stmt} = \text{if } e \text{ then } [s]_{Stmt} \text{ else skip}$	$[\text{INSERT AFTER } p \text{ VALUE } e]_{Upd} = [p]_{Path}(\text{right}[\text{insert } e])$
$[s_1; s_2]_{Stmt} = [s_1]_{Stmt}; [s_2]_{Stmt}$	$[\text{INSERT AS LAST INTO } p \text{ VALUE } e]_{Upd} = [p]_{Path}(\text{children}[\text{left}[\text{insert } e]])$
$[\text{LET } x = e \text{ IN } s]_{Stmt} = \text{let } x = e \text{ in } [s]_{Stmt}$	$[\text{INSERT AS FIRST INTO } p \text{ VALUE } e]_{Upd} = [p]_{Path}(\text{children}[\text{right}[\text{insert } e]])$
$[.]_{Path}(s) = s$	$[\text{DELETE } p]_{Upd} = [p]_{Path}(\text{delete})$
$[p/p']_{Path}(s) = [p]_{Path}([p']_{Path}(s))$	$[\text{DELETE FROM } p]_{Upd} = [p]_{Path}(\text{children}[\text{delete}])$
$[\phi]_{Path}(s) = \text{children}[\text{iter}[\phi?s]]$	$[\text{RENAME } p \text{ TO } n]_{Upd} = [p]_{Path}(\text{rename } n)$
$[p[e]]_{Path}(s) = [p]_{Path}(\text{if } e \text{ then } s \text{ else skip})$	$[\text{REPLACE } p \text{ WITH } e]_{Upd} = [p]_{Path}(\text{delete; insert } e)$
$[x \text{ AS } p]_{Path}(s) = [p]_{Path}(\text{snapshot } x \text{ in } s)$	$[\text{REPLACE IN } p \text{ WITH } e]_{Upd} = [p]_{Path}(\text{children}[\text{delete; insert } e])$
	$[\text{UPDATE } p \text{ BY } s]_{Upd} = [p]_{Path}([s]_{Stmt})$

Figure 5. Source update normalization

$\Gamma \vdash_{Stmt} \{\tau\} s \{\tau'\}$
$\frac{\Gamma \vdash_{Stmt} \{\tau\} s_1 \{\tau'\} \quad \Gamma \vdash_{Stmt} \{\tau'\} s_2 \{\tau''\}}{\Gamma \vdash_{Stmt} \{\tau\} s_1; s_2 \{\tau''\}}$
$\frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash_{Stmt} \{\alpha\} s \{\tau'\}}{\Gamma \vdash_{Stmt} \{\tau\} \text{IF } e \text{ THEN } s \{\tau \tau'\}}$
$\frac{\Gamma \vdash e : \tau_0 \quad \Gamma, x : \tau +_0 \vdash_{Stmt} \{\tau\} s \{\tau'\}}{\Gamma \vdash_{Stmt} \{\tau\} \text{LET } x = e \text{ IN } s \{\tau'\}}$
$\frac{\Gamma \vdash_{Upd} \{\alpha\} u \{\alpha'\}}{\Gamma \vdash_{Stmt} \{\alpha\} u \{\alpha'\}}$

Figure 6. Typechecking rules for compound updates

XQuery normalization rules (Draper et al. 2007). The problem of typechecking unnormalized XQuery expressions is an orthogonal issue (and one that has to our knowledge not been addressed).

Typechecking source-level updates is challenging because simple updates may change the types of many parts of the document simultaneously, depending on the structure of the path p . In contrast, core FLUX updates are easy to typecheck because they break the corresponding navigation, selection, and modification of types into small, manageable steps.

To deal with the non-local nature of source updates, we employ *type variables* Z and *context-tagged type substitutions* Θ . These substitutions are defined as follows:

$$\Theta ::= \emptyset \mid \Theta, Z \mapsto (\Gamma \triangleright \tau)$$

We distinguish the type variables Z we will use here for typechecking source updates from the type variables X used in recursive type definitions E ; we refer to the latter as *defined type variables*. We require the bindings Z in Θ to be unique.

We often treat substitutions Θ as sets or finite maps and in particular write $\Theta \uplus \Theta'$ for the context-tagged substitution resulting from taking the union of the bindings in Θ and Θ' , provided their domains are disjoint. We also write $\tau(\Theta)$ for the result of replacing each occurrence of an undefined type variable in τ with its binding in Θ . Moreover, we write $\Theta(\Theta')$ for the result of applying Θ' to each type in Θ , again ignoring contexts. Substitution application ignores the contexts Γ ; they are only used to typecheck updates within the scope of a path. We consider the free type variables of Θ to be the free variables of Γ and τ in the bindings $\Gamma \triangleright \tau$.

To typecheck a simple update such as $\text{DELETE } a/b$ against an atomic type such as $a[b[], c[]]$, we proceed as follows:

1. First *match* p against the input type, and *split* the type α of the document into a pair (α', Θ) such that $\alpha = \alpha'(\Theta)$.

For example, $a[b[], c[]] = a[Z, c[]](Z \mapsto \cdot \triangleright b[])$.

2. Next *modify* Θ according to the update operation to obtain Θ' . For this we use the Core FLUX type system to update each binding in Θ . This is only a convenience.

Continuing the example, for a deletion we update $Z \mapsto \cdot \triangleright b[]$ to $Z \mapsto \cdot \triangleright ()$.

3. Finally, *apply* Θ' to α' to get the desired final type after the update.

For example, applying $a[Z, c[]](Z \mapsto \cdot \triangleright ())$ we get $a[], c[]$, as desired (this is equivalent to $a[c[]]$).

Figures 7 and 8 show selected typechecking judgments for simple updates and paths. We introduce auxiliary judgments such as path filter checking ($\Gamma \vdash \tau :: \phi \rightsquigarrow (\tau', \Theta)$, shown in Figure 9), simultaneous core statement checking ($\vdash^1 \{\Theta\} s \{\Theta'\}$), and simultaneous path checking ($\vdash_{Path} \{\Theta\} p \rightsquigarrow (\Theta', \Theta'')$). The

For many of the typechecking judgments, we also need to typecheck an expression against all of the bindings of a context-tagged substitution Θ . We therefore introduce several *simultaneous typechecking* judgments. The full system, including the (straightforward) compound statement typechecking judgment $\Gamma \vdash_{Stmt} \{\alpha\} s \{\alpha'\}$ and all auxiliary judgments, is shown in the appendix.

The simple update typechecking rules each follow the procedure outlined above. The path typechecking rules match p against the input type α as described in step 2 above. Note that paths may bind variables, and the same variable may be bound to different types in different cases; this is why we need to include contexts Γ in each binding of the substitutions Θ .

The following operation $\Theta \oplus x$ is used to typecheck $x \text{ AS } p$; it adds the binding $x : \tau$ to each binding $\Gamma \triangleright \tau$ in Θ .

$$\begin{aligned} \emptyset \oplus x &= \emptyset \\ (\Theta, Z \mapsto (\Gamma \triangleright \tau)) \oplus x &= \Theta \oplus x, Z \mapsto (\Gamma, x : \tau \triangleright \tau) \end{aligned}$$

The typechecking rule for conditional paths $p[e]$ is slightly subtle. After typechecking p , we obtain a pair (α', Θ) that splits α into an unchanged part α' and a substitution Θ showing where changes may occur. Since we do not know whether e will hold, we must adjust α' by replacing each occurrence of a variable Z with $Z|\Theta(Z)$. This is accomplished using the substitution $\Theta?$:

$$\begin{aligned} \emptyset? &= \emptyset \\ \Theta, Z \mapsto (\Gamma \triangleright \alpha)? &= \Theta?, Z \mapsto (\Gamma \triangleright Z|\alpha) \end{aligned}$$

5.2 Metatheory

Whenever we translate between two typed languages, we would like to know whether the translation is *sound* (i.e., *type-preserving*). This ensures that if we typecheck the expression in the source language then its translation will also typecheck. ⁵ Conversely, if

⁵In an implementation, one often wants to re-typecheck after translation anyway as a sanity check for the translator.

$$\boxed{\Gamma \vdash_{Upd} \{\alpha\} u \{\alpha'\}}$$

$$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{left}[\text{insert } e] \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{INSERT BEFORE } p \text{ VALUE } e \{\alpha' \langle \Theta' \rangle\}}$$

$$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{children}[\text{left}[\text{insert } e]] \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{INSERT AS LAST INTO } p \text{ VALUE } e \{\alpha' \langle \Theta' \rangle\}}$$

$$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{delete} \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{DELETE } p \{\alpha' \langle \Theta' \rangle\}}$$

Figure 7. Selected typechecking rules for simple updates

$$\boxed{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta)}$$

$$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta_1) \quad \vdash_{Path} \{\Theta_1\} p' \rightsquigarrow (\Theta_2, \Theta_2')}{\Gamma \vdash_{Path} \{\alpha\} p/p' \rightsquigarrow (\alpha' \langle \Theta_2 \rangle, \Theta_2')}$$

$$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash \{\Theta\} e : \text{bool}}{\Gamma \vdash_{Path} \{\alpha\} p[e] \rightsquigarrow (\alpha' \langle \Theta \rangle, \Theta)}$$

$$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \Gamma \vdash \tau :: \phi \rightsquigarrow (\tau', \Theta)}{\Gamma \vdash_{Path} \{\alpha\} p \text{ AS } \tau \rightsquigarrow (\alpha', \Theta \oplus x) \quad \Gamma \vdash_{Path} \{n[\tau]\} \phi \rightsquigarrow (n[\tau'], \Theta)}$$

Figure 8. Typechecking rules for paths

$$\boxed{\Gamma \vdash \tau :: \phi \rightsquigarrow (\tau', \Theta)}$$

$$\frac{\alpha \not\prec: \phi}{\Gamma \vdash () :: \phi \rightsquigarrow ((), \emptyset)} \quad \frac{\alpha \prec: \phi \quad Z \text{ fresh}}{\Gamma \vdash \alpha :: \phi \rightsquigarrow (Z, Z \mapsto (\Gamma \triangleright \alpha))}$$

$$\frac{\Gamma \vdash \tau_1 :: \phi \rightsquigarrow (\tau'_1, \Theta_1) \quad \Gamma \vdash \tau_2 :: \phi \rightsquigarrow (\tau'_2, \Theta_2)}{\Gamma \vdash \tau_1, \tau_2 :: \phi \rightsquigarrow ((\tau'_1, \tau'_2), \Theta_1 \uplus \Theta_2)}$$

$$\frac{\Gamma \vdash \tau_1 :: \phi \rightsquigarrow (\tau'_1, \Theta_1) \quad \Gamma \vdash \tau_2 :: \phi \rightsquigarrow (\tau'_2, \Theta_2)}{\Gamma \vdash \tau_1 | \tau_2 :: \phi \rightsquigarrow (\tau'_1 | \tau'_2, \Theta_1 \uplus \Theta_2)}$$

$$\frac{\Gamma \vdash \tau_1 :: \phi \rightsquigarrow (\tau_2, \Theta) \quad \Gamma \vdash E(X) :: \phi \rightsquigarrow (\tau', \Theta)}{\Gamma \vdash \tau_1^* :: \phi \rightsquigarrow (\tau_2^*, \Theta) \quad \Gamma \vdash X :: \phi \rightsquigarrow (\tau', \Theta)}$$

Figure 9. Typechecking rules for path filters

$$\boxed{\vdash \{\Theta\} e : \tau}$$

$$\frac{\vdash \{\Theta\} e : \tau \quad \Gamma \vdash e : \tau}{\vdash \{\emptyset\} e : \tau \quad \vdash \{\Theta, Z \mapsto (\Gamma \triangleright \alpha)\} e : \tau}$$

$$\boxed{\vdash^1 \{\Theta\} s \{\Theta'\}}$$

$$\frac{\vdash^1 \{\Theta\} s \{\Theta'\} \quad \Gamma \vdash^1 \{\alpha\} s \{\tau\}}{\vdash^1 \{\emptyset\} s \{\emptyset\} \quad \vdash^1 \{\Theta, Z \mapsto (\Gamma \triangleright \alpha)\} s \{\Theta', Z \mapsto (\Gamma \triangleright \tau)\}}$$

$$\boxed{\vdash_{Stmt} \{\Theta\} s \{\Theta'\}}$$

$$\frac{\vdash_{Stmt} \{\Theta\} s \{\Theta'\} \quad \Gamma \vdash_{Stmt} \{\tau\} s \{\tau'\}}{\vdash_{Stmt} \{\emptyset\} s \{\emptyset\} \quad \vdash_{Stmt} \{\Theta, Z \mapsto (\Gamma \triangleright \tau)\} s \{\Theta', Z \mapsto (\Gamma \triangleright \tau')\}}$$

$$\boxed{\vdash_{Path} \{\Theta\} p \rightsquigarrow (\Theta', \Theta'')}$$

$$\frac{\vdash_{Path} \{\emptyset\} p \rightsquigarrow (\emptyset, \emptyset) \quad \vdash_{Path} \{\Theta\} p \rightsquigarrow (\Theta', \Theta'') \quad \Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta''')}{\vdash_{Path} \{\Theta, Z \mapsto (\Gamma \triangleright \alpha)\} p \rightsquigarrow (\Theta', Z \mapsto (\Gamma \triangleright \alpha'), \Theta'' \uplus \Theta''')}$$

Figure 10. Simultaneous typechecking judgments

the source language expression fails to typecheck, it is preferable to report the error in terms of the source language using the source type system. We have established that the translation is indeed sound:

Theorem 4 (Soundness). *Assume Γ, τ, τ' have no free type variables Z . Then if $\Gamma \vdash_{Stmt} \{\tau\} s \{\tau'\}$ then $\Gamma \vdash^* \{\tau\} [s]_{Stmt} \{\tau'\}$.*

Conversely, another concern is that the source-level type system might be too restrictive. Are there source-level expressions whose *translations* are well-formed, but which are not well-formed in the source-level system? This is the question of *completeness*, that is, whether the translation *reflects* typability. If this completeness property did not hold, this would indicate that the source type system could be made more expressive. Fortunately, however, completeness does hold:

Theorem 5 (Completeness). *Assume Γ, τ, τ' have no free type variables Z . Then if $\Gamma \vdash^* \{\tau\} [s]_{Stmt} \{\tau'\}$ then $\Gamma \vdash_{Stmt} \{\tau\} s \{\tau'\}$.*

6. Path-errors and dead-code analysis

Besides developing a type system for μXQ , Colazzo et al. (2006) studied the problem of identifying subexpressions of the query that always evaluate to $()$, but are not syntactically equal to $()$. Such “unproductive” subexpressions typically indicate errors in a query. For example, the query `for $y \in x/a$ return $a[]$` is well-formed in context $\Gamma = x : b[c]^{*}, d[]^{*}$, but unproductive when evaluated against Γ since x/a will always be empty. Colazzo et al. (2006) formally defined such *path-errors*⁶ and introduced a type-based analysis that detects them. In this section, we define path-errors for updates and derive a path-error analysis for core FLUX. We first introduce technical machinery, then define path-errors and the analysis, and prove its correctness.

Consider *locations* l . We will work with *distinctly labeled statements* s_l in which each core FLUX subexpression carries a distinct location l . We ignore locations as convenient when we wish to view s_l as an ordinary statement s . Suppose s is distinctly labeled. We write $s[l]$ for the unique subexpression of e labeled by l and write $s|_l$ for the result of replacing the subexpression at l in s with `skip`. For example, $(\text{iter}[\text{children}[s_l]_{l'}]_{l''})_{l''} = \text{iter}[\text{skip}]$.

We now define a form of path-errors suitable for updates, based on replacing subexpressions with the trivial update `skip` instead of the empty sequence $()$.

Definition 1. Suppose $\Gamma \vdash^a \{\tau\} s \{\tau'\}$, where s is distinctly labeled. We say s is *unproductive* at l provided $\gamma; v \vdash s \Rightarrow^u v' \iff \gamma; v \vdash s|_l \Rightarrow^u v'$ for every $\gamma \in [\Gamma], v \in [\tau], v' \in [\tau']$. Recall that update evaluation is functional so this means that s and $s|_l$ are equivalent over inputs from $[\Gamma], [\tau]$.

Moreover, we say that s has an *update path-error* at l provided s is unproductive at l and $s[l] \neq \text{skip}$, and say that s is *update path-correct* if s has no update path errors.

We define a static analysis for identifying update path-errors via the rules in Figure 11. The main judgment is $\Gamma \vdash^a \{\tau\} s_l \{\tau'\} \& L$, meaning s is *well-formed and is unproductive at each $l \in L$* . We employ an auxiliary judgment $\Gamma \vdash_{\text{iter}} \{\tau\} s_l \{\tau'\} \& L$ to handle iteration. We also define a “conditional union” operation:

$$L[l_1, \dots, l_n \Rightarrow l] = \begin{cases} L \cup \{l\} & \{l_1, \dots, l_n\} \subseteq L \\ L & \text{otherwise} \end{cases}$$

Note that the analysis is intraprocedural. It gives up when we consider a procedure call $P(\bar{e})$: we conservatively assume that

⁶ Arguably, the term “path-errors” is inaccurate in that there are expressions such as `for $\bar{x} \in ()$ return \bar{x}` that do not mention path expressions, yet contain path-errors. Nevertheless, we follow the existing terminology here.

$\Gamma \vdash^a \{\tau\} s \{\tau'\} \& L$			
$\Gamma \vdash^a \{\tau\} \text{skip}_l \{\tau\} \& \{l\}$	$\frac{\Gamma \vdash^a \{\tau\} (s_1)_{l_1} \{\tau'\} \& L_1 \quad \Gamma \vdash^a \{\tau'\} (s_2)_{l_2} \{\tau''\} \& L_2}{\Gamma \vdash^a \{\tau\} ((s_1)_{l_1}; (s_2)_{l_2})_l \{\tau''\} \& (L_1 \cup L_2)[l_1, l_2 \Rightarrow l]}$	$\frac{\Gamma, x:\tau \vdash^a \{\tau\} s_{l'} \{\tau'\} \& L}{\Gamma \vdash^a \{\tau\} (\text{snapshot } x \text{ in } s_{l'})_l \{\tau'\} \& L[l' \Rightarrow l]}$	
$\frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash^a \{\tau\} (s_1)_{l_1} \{\tau_1\} \& L_1 \quad \Gamma \vdash^a \{\tau\} (s_2)_{l_2} \{\tau_2\} \& L_2}{\Gamma \vdash^a \{\tau\} (\text{if } e \text{ then } (s_1)_{l_1} \text{ else } (s_2)_{l_2})_l \{\tau_1\} \& (L_1 \cup L_2)[l_1, l_2 \Rightarrow l]}$	$\frac{\Gamma \vdash e : \tau \quad \Gamma, x:\tau \vdash^a \{\tau_1\} s \{\tau_2\} \& L}{\Gamma \vdash^a \{\tau_1\} (\text{let } x = e \text{ in } s_{l'})_l \{\tau_2\} \& L[l' \Rightarrow l]}$		
$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash^* \{\tau\} (\text{insert } e)_l \{\tau\} \& \{l \mid \tau <: ()\}}$	$\frac{}{\Gamma \vdash^a \{\tau\} \text{delete}_l \{\tau\} \& \{l \mid \tau <: ()\}}$	$\frac{}{\Gamma \vdash^1 \{n'[\tau]\} (\text{rename } n)_l \{n[\tau]\} \& \{l \mid n = n'\}}$	
$\frac{\alpha <: \phi \quad \Gamma \vdash^1 \{\alpha\} s_{l'} \{\tau\} \& L}{\Gamma \vdash^1 \{\alpha\} (\phi? s_{l'})_l \{\tau\} \& L[l' \Rightarrow l]}$	$\frac{\alpha \not<: \phi}{\Gamma \vdash^1 \{\alpha\} (\phi? s_{l'})_l \{\alpha\} \& \{l\}}$	$\frac{\Gamma \vdash_{\text{iter}} \{\tau\} s_{l'} \{\tau'\} \& L}{\Gamma \vdash^* \{\tau\} \text{iter}_{[s_{l'}]_l} \{\tau'\} \& L[l' \Rightarrow l]}$	$\frac{\Gamma \vdash^a \{\tau\} P(\vec{e}) \{\tau'\}}{\Gamma \vdash^a \{\tau\} P(\vec{e}) \{\tau'\} \& \emptyset}$
$\frac{\Gamma \vdash^* \{\tau\} s_{l'} \{\tau'\} \& L}{\Gamma \vdash^1 \{n[\tau]\} \text{children}_{[s_{l'}]_l} \{n[\tau']\} \& L[l' \Rightarrow l]}$	$\frac{\Gamma \vdash^* \{\tau\} \{\tau'\} \& L}{\Gamma \vdash^a \{\tau\} \text{left}_{[s_{l'}]_l} \{\tau', \tau\} \& L[l' \Rightarrow l]}$	$\frac{\Gamma \vdash^* \{\tau\} \{\tau'\} \& L}{\Gamma \vdash^a \{\tau\} \text{right}_{[s_{l'}]_l} \{\tau, \tau'\} \& L[l' \Rightarrow l]}$	
$\Gamma \vdash_{\text{iter}} \{\tau\} s \{\tau'\} \& L$			
$\frac{}{\Gamma \vdash_{\text{iter}} \{\tau\} s \{\tau'\} \& L}$	$\frac{\Gamma \vdash^1 \{\alpha\} s \{\tau\} \& L}{\Gamma \vdash_{\text{iter}} \{\alpha\} s \{\tau\} \& L}$	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau_2\} \& L}{\Gamma \vdash_{\text{iter}} \{\tau_1^*\} s \{\tau_2^*\} \& L}$	$\frac{\Gamma \vdash_{\text{iter}} \{E(X)\} s \{\tau\} \& L}{\Gamma \vdash_{\text{iter}} \{X\} s \{\tau\} \& L}$
$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1, \tau_2\} s \{\tau'_1, \tau'_2\} \& L_1 \cap L_2}$	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
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	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
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	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1 \quad \Gamma \vdash_{\text{iter}} \{\tau_2\} s \{\tau'_2\} \& L_2}{\Gamma \vdash_{\text{iter}} \{\tau_1\} s \{\tau'_1\} \& L_1}$		
	$\frac{\Gamma \vdash_{\text{iter}} \{\tau_1\}$		

Figure 11. Path-error analysis for updates

there is no path error at $P(\vec{e})$, and we do not proceed to analyze the body of P . We can, of course, extend the analysis to declarations Δ by analyzing each procedure body individually.

We first establish that the analysis produces results whenever s is well-formed. This is straightforward by induction on derivations.

Lemma 1.

1. If $\Gamma \vdash^a \{\tau\} s \{\tau'\}$ then there exists L such that $\Gamma \vdash^a \{\tau\} s \{\tau'\} \& L$.
2. If $\Gamma \vdash_{\text{iter}} \{\tau\} s \{\tau'\}$ then there exists L such that $\Gamma \vdash_{\text{iter}} \{\tau\} s \{\tau'\} \& L$.

The goal of the analysis is to conservatively *underestimate* the set of possible unproductive locations in s .

Theorem 6 (Path-Error Analysis Soundness).

1. If $\Gamma \vdash^a \{\tau\} s \{\tau'\} \& L$ then s is unproductive at every $l \in L$.
2. If $\Gamma \vdash_{\text{iter}} \{\tau\} s \{\tau'\} \& L$ then $\text{iter}[s]$ is unproductive at every $l \in L$.

Moreover, all of the labels in the set $\{l \in L \mid s[l] \neq \text{skip}\}$ can be reported as update path-errors and used to optimize s by replacing each $s[l]$ with skip .

Using more sophisticated rules for typechecking let - and for -expressions, (Colazzo et al. 2006) were also able to show that their path-error analysis is *complete* for μXQ (without recursion); thus, path-correctness is decidable for the fragment of XQuery they studied — a nontrivial result. Similar techniques can be used to make update path-error analysis more precise, but it is not obvious that this yields a complete analysis, even in the absence of recursion. We leave this issue for future work.

It is, of course, also of interest to perform path-error analysis at the source level, so that the errors can be reported in terms familiar to the user. We believe that the path-error analysis can be “lifted” to the source type system, but leave this for future work. However, it appears that many path-errors show up in the source type system as empty substitutions Θ resulting from analyzing path expressions.

7. Related work

Other database update languages Liefke and Davidson (1999)’s update language CPL+, a typed language for updating complex-object databases using path-based insert, update, and delete operations. High-level CPL+ updates were translated to a simpler core language with orthogonal operations for iteration, navigation, insertion/deletion, and replacement. The FLUX core language was strongly influenced by CPL+.

Static typing for XML processing We will focus on only the most closely related work on XML typechecking; Møller and Schwartzbach (2005) provide a much more complete survey of type systems for XML transformation languages.

Hosoya et al. (2005) introduced XDuce, the first statically typed XML transformation language based on regular expressions. Fernandez et al. (2001) introduced many of the ideas for using XDuce-style regular expression subtyping for typechecking an XML query language based on monadic comprehensions. XQuery’s type system (Draper et al. 2007) is also based on regular expression types and subtyping, but its rules for typechecking iteration are relatively imprecise: they discard information about the order and multiplicity of the elements of a sequence. As discussed by Cheney (2008), taking this approach to typechecking iterations in *updates* would be disastrous since many updates iterate over a part of the database while leaving its structure intact. Colazzo et al. (2006) showed how to provide more precise regular expression types to XQuery *for*-iteration; we have already discussed this work in the body of the paper. Cheney (2008) showed how to add subtyping and subsumption to μXQ and FLUX while retaining decidable typechecking.

XML update languages (Cheney 2007) provided a detailed discussion of XML update language proposals and compared them with the FLUX approach. Here, we will only discuss closely related or more recent work.

Calcagno et al. (2005) investigated DOM-style XML updates using *context logic*, a logic of “trees with holes”. Calcagno et al. (2005) studied a Hoare-style logic for sequences of atomic update operations on unordered XML. Gardner et al. (2008) extended this approach to ordered XML and while-programs over atomic DOM updates. This approach is very promising for reasoning about low-

level DOM updates, for example in Java or JavaScript programs. It should be possible to translate core FLUX to their variant of DOM; it would be interesting to see whether FLUX type information can also be compiled down to context logic in an appropriate way.

The W3C XQuery Update Facility (Chamberlin et al. 2008) has been under development for several years. However, the typing rules in the current draft treat updates as expressions of type $()$, and to our knowledge this type system has not been proved sound.

Ghelli et al. (2007b) have developed XQueryU, a variant of XQuery! that is translated to an “algebraic” core language intended for optimization. However, the semantics of the core language is defined by translation back to XQueryU, which seems circular.

Static typechecking has not been studied for any other extant XML update language proposals, even though the W3C’s XQuery Update Facility Requirements document (Chamberlin and Robie 2005) lists static typechecking as a strong requirement. FLUX shows that applying well-known functional language design principles leads to a language with a relatively simple semantics and relatively straightforward type system.

Static analysis techniques have been studied for only a few of these languages. Benedikt et al. (2005a) and Benedikt et al. (2005b) studied static analysis techniques for optimizing updates in UpdateX, an earlier XML update language proposal due to Sur et al. (2004). Ghelli et al. (2007a) have developed a commutativity analysis for determining when two side-effecting expressions in XQuery! can be reordered. No prior work has addressed path-error or dead code analysis for XML updates.

The design goals of many of these proposals differ from those that motivate this work. FLUX is not meant to be a full-fledged programming language for mutable XML data. Instead, it is meant to play a role for XML and XQuery similar to that of SQL’s update facilities relative to relational databases and SQL. Its goal is only to be expressive enough for typical updates to XML databases while remaining simple and statically typecheckable.

Mutability in functional languages FLUX takes a “purely functional” approach to typechecking updates. The type of an update reflects the changes to the mutable store an update may make. This is similar to side-effect encapsulation using monads or arrows in Haskell. An alternative possibility might be to use ML-like references. This could easily handle updates to parts of an XML database whose type is fixed; however, handling updates that change the *type* of a part of the database would likely be problematic, due to aliasing issues. FLUX does not allow aliasing of the mutable store, so avoids this problem.

8. Extensions and future work

Additional XQuery features To simplify the discussion, we have omitted features such as attributes, comments, and processing instructions that are present in the official XQuery data model, as well as the XPath axes needed to access them. We have also omitted the many additional base types and built-in functions (such as `position()` or `last()`) present in full XQuery/XPath. All of these features can be added without damaging the formal properties of the core language.

We have also omitted the descendant axis. Many DTDs and XML Schemas encountered in database applications of XML are nonrecursive and “shallow” (Choi 2002; Bex et al. 2004). Thus, in practice, vertical recursion (the descendant axis `//`) can usually be avoided. Simple updates involving `//` can, however, be simulated using recursive update procedures; in fact, for non-recursive input types, updates involving `//` can often already be expressed in FLUX. Further work is needed to understand the expressiveness and usability tradeoffs involved in typechecking more complex updates involving recursive types.

Transformations The XQuery Update Facility (Chamberlin et al. 2008) includes *transformations*, which allow running an update operation within an XQuery expression, with side-effects confined (somewhat like `runST` in Haskell). Such a facility can easily be added using FLUX updates:

$$e ::= \dots \mid \text{transform } e \text{ by } s$$

with semantics and typing rules:

$$\frac{\gamma \vdash e \Rightarrow v \quad \gamma; v \vdash s \Rightarrow^U v'}{\gamma \vdash \text{transform } e \text{ by } s \Rightarrow v'} \quad \frac{\Gamma \vdash e : \tau_1 \quad \Gamma \vdash^* \{ \tau_1 \} s \{ \tau_2 \}}{\Gamma \vdash \text{transform } e \text{ by } s : \tau_2}$$

Dynamic typechecking, incremental validation and maintenance

We believe it is important to combine FLUX-style static typechecking with efficient dynamic techniques in order to handle cases where static type information is imprecise. Barbosa et al. (2004) and Balmin et al. (2004) have studied efficient *incremental validation* techniques for checking that sequences of atomic updates preserve a database’s schema. These techniques impose (manageable, but nonzero) run-time costs per atomic update operation and storage overhead proportional to the database size; also, they require that the input and output types are equal, a significant limitation compared to FLUX.

Efficient implementation within XML databases We have built a prototype FLUX interpreter in OCaml, in order to validate our type system and normalization translation designs and experiment with variations. The obvious next step is developing efficient implementations of FLUX, particularly within XML database systems. Liefke and Davidson (1999) investigated efficient implementation techniques for CPL+ updates to complex-object databases, which have much in common with XML databases. One initial implementation strategy could simply be to generate XQuery! or XQuery Updates from core FLUX after normalization, typechecking and high-level optimization; this should not be difficult since these languages are more expressive than FLUX. However, more sophisticated techniques may be necessary to obtain good performance.

9. Conclusions

The problem of updating XML databases poses many challenges to language design. In previous work, we introduced FLUX, a simple core language for XML updates, inspired in large part by the language CPL+ introduced by Liefke and Davidson (1999) for updating complex object databases. In contrast to all other update proposals of which we are aware, FLUX preserves the good features of XQuery such as its purely functional semantics, while offering features convenient for updating XML. Moreover, FLUX is the first proposal for updating XML to be equipped with a sound, static type system.

In this paper we have further developed the foundations of FLUX, relaxing the limitations present in our preliminary proposal. First, we have extended its operational semantics and type system to handle *recursive types and updates*. This turned out to be straightforward. Second, although the FLUX core language is easy to understand, typecheck and optimize, it is not easy to use. Therefore, we have developed a *high-level source language* for updates, and shown how to translate it to core FLUX. Since it is difficult to propagate useful type error information from translated updates back to source updates, we have also developed a type system for the source language, and validated its design by proving that the translation both *preserves* and *reflects* typability. Third, we developed a novel definition of *update path-errors*, a form of dead code analysis, and introduced a static analysis that identifies them.

At present we have implemented a proof-of-concept prototype FLUX interpreter, including typechecking for the source language and core language, normalization, and path-error analysis. There

are many possible directions for future work; the most immediate is to develop efficient optimizing implementations of FLUX within existing XML databases or other XML-processing systems.

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A. Semantics and type system for μXQ queries

We will use an XQuery-like core language called μXQ , introduced by Colazzo et al. (2006). Following that paper, we distinguish between *tree variables* $\bar{x} \in TVar$, introduced by `for`, and *forest variables*, $x \in Var$, introduced by `let`. The other syntactic classes of our variant of μXQ include labels $l, m, n \in Lab$ and expressions $e \in Expr$; the abstract syntax of expressions is defined by the following BNF grammar:

$$\begin{aligned} e ::= & () \mid e, e' \mid n[e] \mid w \mid x \mid \text{let } x = e \text{ in } e' \\ & \mid \text{true} \mid \text{false} \mid \text{if } c \text{ then } e \text{ else } e' \mid e \approx e' \\ & \mid \bar{x} \mid \bar{x}/\text{child} \mid e :: n \mid \text{for } \bar{x} \in e \text{ return } e' \end{aligned}$$

The distinguished variables \bar{x} in `for $\bar{x} \in e$ return $e'(\bar{x})$` and x in `let $x = e$ in $e''(x)$` are bound in $e'(\bar{x})$ and $e''(x)$ respectively. Here and elsewhere, we employ common conventions such as considering expressions containing bound variables equivalent up to α -renaming and employing a richer concrete syntax including, for example, parentheses.

Recursive queries can be added to μXQ in the same manner as in XQuery without damaging the properties of the system needed in this paper.

To simplify the presentation, we split μXQ 's projection operation $\bar{x} \text{ child} :: l$ into two expressions: child projection (\bar{x}/child) which returns the children of \bar{x} , and node name filtering ($e :: n$) which evaluates e to an arbitrary sequence and selects the nodes labeled n . Thus, the ordinary child axis expression $\bar{x} \text{ child} :: n$ is syntactic sugar for $(\bar{x}/\text{child}) :: n$ and the “wildcard” child axis is definable as $\bar{x} \text{ child} :: * = \bar{x}/\text{child}$. We also consider only one built-in operation, string equality.

B. Type soundness for updates

Type soundness for updates relies on pre-existing results for type soundness for queries, which we repeat here:

Theorem 7 (Query soundness). *If $\Gamma \vdash e : \tau$ and $\gamma \in [\Gamma]$ then $\gamma \vdash e \Rightarrow v$ with $v \in [\tau]$.*

We need the following lemmas summarizing properties of test subtyping and of the operational behavior of iteration.

Lemma 2 (Subtyping and tests). *If $t \in [\alpha]$ then $\alpha <: \phi$ if and only if $t \in [\phi]$.*

Proof. First note that if $\tau \in [\alpha]$ and $\alpha <: \phi$ then $t \in [\phi]$. For the reverse direction, suppose $\tau \in [\alpha]$ and $\alpha \not<: \phi$, and consider all combinations of cases. \square

Lemma 3 (Iteration). *If $\gamma; v_1 \vdash \text{iter}[s] \Rightarrow^u v'_1$, $\gamma; v_2 \vdash \text{iter}[s] \Rightarrow^u v'_2$ are derivable then $\gamma; v_1, v_2 \vdash \text{iter}[s] \Rightarrow^u v'_1, v'_2$ is derivable.*

Theorem 8 (Update soundness).

1. If $\Gamma \vdash^a \{ \tau \} s \{ \tau' \}$, $v \in [\tau]$, and $\gamma \in [\Gamma]$, then $\gamma; v \vdash s \Rightarrow^u v'$ implies $v' \in [\tau']$.
2. If $\Gamma \vdash_{\text{iter}} \{ \tau \} s \{ \tau' \}$, $v \in [\tau]$, and $\gamma \in [\Gamma]$, then $\gamma; v \vdash \text{iter}[s] \Rightarrow^u v'$ implies $v' \in [\tau']$.

Proof. Parts (1) and (2) must be proved simultaneously by induction; in each case the induction is on the typing derivation. The cases involving standard constructs (if, let, skip, sequencing) are omitted. For each case, we first show the typing derivation and then the (unique) corresponding operational derivation that can be constructed, with remarks as appropriate.

$$\begin{aligned} \text{children}(n[f]) &= f \\ \text{children}(v) &= () \quad (v \not\approx n[v']) \\ \llbracket \text{true} \rrbracket \gamma &= \text{true} & \llbracket \text{false} \rrbracket \gamma &= \text{false} \\ \llbracket () \rrbracket \gamma &= () & \llbracket e, e' \rrbracket \gamma &= \llbracket e \rrbracket \gamma, \llbracket e' \rrbracket \gamma \\ \llbracket n[e] \rrbracket \gamma &= n[\llbracket e \rrbracket \gamma] & \llbracket w \rrbracket \gamma &= w \\ \llbracket x \rrbracket \gamma &= \gamma(x) & \llbracket \bar{x} \rrbracket \gamma &= \gamma(\bar{x}) \\ \llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket \gamma &= \llbracket e_2 \rrbracket \gamma[x := \llbracket e_1 \rrbracket \gamma] \\ \llbracket \text{if } c \text{ then } e_1 \text{ else } e_2 \rrbracket \gamma &= \begin{cases} \llbracket e_1 \rrbracket \gamma & \llbracket c \rrbracket \gamma \approx \text{true} \\ \llbracket e_2 \rrbracket \gamma & \llbracket c \rrbracket \gamma \approx \text{false} \end{cases} \\ \llbracket e = e' \rrbracket \gamma &= \begin{cases} \text{true} & \llbracket e \rrbracket \gamma \approx \llbracket e' \rrbracket \gamma \\ \text{false} & \llbracket e \rrbracket \gamma \not\approx \llbracket e' \rrbracket \gamma \end{cases} \\ \llbracket e :: n \rrbracket \gamma &= [n[v] \mid n[v] \in \llbracket e \rrbracket \gamma] \\ \llbracket \bar{x}/\text{child} \rrbracket \gamma &= \text{children}(\gamma(\bar{x})) \\ \llbracket \text{for } \bar{x} \in e_1 \text{ return } e_2 \rrbracket \gamma &= \llbracket e_2 \rrbracket \gamma[\bar{x} := t] \mid t \in \llbracket e_1 \rrbracket \gamma \end{aligned}$$

Figure 12. Semantics of query expressions.

$$\begin{aligned} &\boxed{\Gamma \vdash e : \tau} \\ &\frac{}{\Gamma \vdash () : ()} \quad \frac{}{\Gamma \vdash w : \text{string}} \\ &\frac{}{\Gamma \vdash \text{true} : \text{bool}} \quad \frac{}{\Gamma \vdash \text{false} : \text{bool}} \\ &\frac{\Gamma \vdash e : \tau \quad \Gamma \vdash e' : \tau'}{\Gamma \vdash e, e' : \tau, \tau'} \quad \frac{\Gamma \vdash e : \tau \quad \Gamma \vdash e, e' : \text{string}}{\Gamma \vdash n[e] : n[\tau]} \quad \frac{}{\Gamma \vdash e = e' : \text{bool}} \\ &\frac{\bar{x} : \alpha \in \Gamma \quad x : \tau \in \Gamma \quad \Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \bar{x} : \alpha \quad \Gamma \vdash x : \tau \quad \Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2} \\ &\frac{\Gamma \vdash c : \text{bool} \quad \Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \text{if } c \text{ then } e_1 \text{ else } e_2 : \tau_1 | \tau_2} \\ &\frac{\bar{x} : n[\tau] \in \Gamma \quad \Gamma \vdash e : \tau \quad \tau :: n \Rightarrow \tau'}{\Gamma \vdash \bar{x}/\text{child} : \tau \quad \Gamma \vdash e :: n : \tau'} \\ &\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash \bar{x} \text{ in } \tau_1 \rightarrow e_2 : \tau_2 \quad \Gamma \vdash e : \tau \quad \tau <: \tau'}{\Gamma \vdash \text{for } \bar{x} \in e_1 \text{ return } e_2 : \tau_2 \quad \Gamma \vdash e : \tau'} \\ &\boxed{\tau :: n \Rightarrow \tau'} \\ &\frac{}{n[\tau] :: n \Rightarrow n[\tau]} \quad \frac{\alpha \neq n[\tau]}{\alpha :: n \Rightarrow ()} \quad \frac{}{() :: n \Rightarrow ()} \quad \frac{\tau_1 :: n \Rightarrow \tau_2}{\tau_1^* :: n \Rightarrow \tau_2^*} \\ &\frac{\tau_1 :: n \Rightarrow \tau'_1 \quad \tau_2 :: n \Rightarrow \tau'_2}{\tau_1, \tau_2 :: n \Rightarrow \tau'_1, \tau'_2} \quad \frac{\tau_1 :: n \Rightarrow \tau'_1 \quad \tau_2 :: n \Rightarrow \tau'_2}{\tau_1 | \tau_2 :: n \Rightarrow \tau'_1 | \tau'_2} \\ &\boxed{\Gamma \vdash \bar{x} \text{ in } \tau \rightarrow e : \tau'} \\ &\frac{}{\Gamma \vdash \bar{x} \text{ in } () \rightarrow e : ()} \quad \frac{\Gamma, \bar{x} : \alpha \vdash e : \tau}{\Gamma \vdash \bar{x} \text{ in } \alpha \rightarrow e : \tau} \quad \frac{\Gamma \vdash \bar{x} \text{ in } \tau_1 \rightarrow e : \tau_2}{\Gamma \vdash \bar{x} \text{ in } \tau_1^* \rightarrow e : \tau_2^*} \\ &\frac{\Gamma \vdash \bar{x} \text{ in } \tau_1 \rightarrow e : \tau'_1 \quad \Gamma \vdash \bar{x} \text{ in } \tau_2 \rightarrow e : \tau'_2}{\Gamma \vdash \bar{x} \text{ in } \tau_1, \tau_2 \rightarrow e : \tau'_1, \tau'_2} \\ &\frac{\Gamma \vdash \bar{x} \text{ in } \tau_1 \rightarrow e : \tau'_1 \quad \Gamma \vdash \bar{x} \text{ in } \tau_2 \rightarrow e : \tau'_2}{\Gamma \vdash \bar{x} \text{ in } \tau_1 | \tau_2 \rightarrow e : \tau'_1 | \tau'_2} \end{aligned}$$

Figure 13. Query well-formedness.

- Case (insert):

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash^* \{\ () \} \text{insert } e \{ \tau \}} \quad \frac{\gamma \vdash e \Rightarrow v}{\gamma; () \vdash \text{insert } e \Rightarrow^U v}$$

Follows by query soundness (Theorem 2).

- Case (delete):

$$\frac{}{\Gamma \vdash^a \{ \tau \} \text{delete } \{ () \}} \quad \frac{}{\gamma; v \vdash \text{delete} \Rightarrow^U ()}$$

Immediate.

- Case (rename):

$$\frac{}{\Gamma \vdash^* \{ n'[\tau] \} \text{rename } n \{ n[\tau] \}} \quad \frac{}{\gamma; n'[v] \vdash \text{rename } n \Rightarrow^U n[v]}$$

Follows since $n'[v] \in \llbracket n'[\tau] \rrbracket$ implies $v \in \llbracket \tau \rrbracket$ so $n[v] \in \llbracket n[\tau] \rrbracket$.

- Case (snapshot):

$$\frac{\Gamma, x : \tau \vdash^a \{ \tau \} s \{ \tau' \}}{\Gamma \vdash^a \{ \tau \} \text{snapshot } x \text{ in } s \{ \tau' \}} \quad \frac{\gamma[x := v]; v \vdash s \Rightarrow^U v'}{\gamma; v \vdash \text{snapshot } x \text{ in } s \Rightarrow^U v'}$$

Follows by induction, using the fact that $v \in \llbracket \tau \rrbracket$ so that $\gamma[x := v] \in \llbracket \Gamma, x : \tau \rrbracket$.

- Case (test1):

$$\frac{\alpha <: \phi \quad \Gamma \vdash^1 \{ \alpha \} s \{ \tau \}}{\Gamma \vdash^1 \{ \alpha \} \phi?s \{ \tau \}} \quad \frac{t \in \llbracket \phi \rrbracket \quad \gamma; t \vdash s \Rightarrow^U v}{\gamma; t \vdash \phi?s \Rightarrow^U v}$$

This case follows immediately by appealing to Lemma 2 and then the induction hypothesis.

- Case (test2):

$$\frac{\alpha \not<: \phi}{\Gamma \vdash^1 \{ \alpha \} \phi?s \{ \alpha \}} \quad \frac{t \notin \llbracket \phi \rrbracket}{\gamma; t \vdash \phi?s \Rightarrow^U t}$$

This case is immediate by Lemma 2.

- Case (children):

$$\frac{\Gamma \vdash^* \{ \tau \} s \{ \tau' \}}{\Gamma \vdash^1 \{ n[\tau] \} \text{children}[s] \{ n[\tau'] \}}$$

$$\frac{\gamma; v \vdash s \Rightarrow^U v'}{\gamma; n[v] \vdash \text{children}[s] \Rightarrow^U n[v']}$$

Clearly, since $n[v] \in \llbracket n[\tau] \rrbracket$, we must have $v \in \llbracket \tau \rrbracket$. By induction, we have that $v' \in \llbracket \tau' \rrbracket$, from which it is immediate that $n[v'] \in \llbracket n[\tau'] \rrbracket$.

- Case (right):

$$\frac{\Gamma \vdash^* \{ () \} s \{ \tau' \}}{\Gamma \vdash^a \{ \tau \} \text{right}[s] \{ \tau, \tau' \}} \quad \frac{\gamma; () \vdash s \Rightarrow^U v'}{\gamma; v \vdash \text{right}[s] \Rightarrow^U v, v'}$$

By assumption, $v \in \llbracket \tau \rrbracket$, induction, we have that $v' \in \llbracket \tau' \rrbracket$, so $v, v' \in \llbracket \tau, \tau' \rrbracket$.

- Case (left): Symmetric.

- Case (iter):

$$\frac{\Gamma \vdash_{\text{iter}} \{ \tau \} s \{ \tau' \}}{\Gamma \vdash^* \{ \tau \} \text{iter}[s] \{ \tau' \}}$$

We proceed using induction hypothesis (3).

- Case $P(\vec{e})$: Suppose the typing derivation is of the form

$$\frac{\begin{array}{c} P(\vec{x} : \vec{\tau}) : \sigma_1 \Rightarrow \sigma_2 \triangleq s \in \Delta \quad \sigma_1' <: \sigma_1 \\ \Gamma \vdash e_1 : \tau_1' \quad \tau_1' <: \tau_1 \\ \dots \\ \Gamma \vdash e_n : \tau_n' \quad \tau_n' <: \tau_n \end{array}}{\Gamma \vdash^a \{ \sigma_1' \} P(\vec{e}) \{ \sigma_2 \}}$$

Hence the operational semantics derivation must be of the form:

$$\frac{\begin{array}{c} P(\vec{x} : \vec{\tau}) : \sigma_1 \Rightarrow \sigma_2 \triangleq s \in \Delta \\ \gamma \vdash e_1 \Rightarrow v_1 \\ \dots \\ \gamma \vdash e_n \Rightarrow v_n \\ \gamma[x_1 := v_1, \dots, x_n := v_n]; v \vdash s \Rightarrow^U v' \end{array}}{\gamma; v \vdash P(\vec{e}) \Rightarrow^U v'}$$

Then by query soundness and the definition of $<$: we have $v_i \in \llbracket \tau_i' \rrbracket \subseteq \llbracket \tau_i \rrbracket$ for each $i \in \{1, \dots, n\}$. Hence, $\gamma[x_1 := v_1, \dots, x_n := v_n] \in \llbracket \Gamma, x_1 : \tau_1, \dots, x_n : \tau_n \rrbracket$. Moreover, $v \in \llbracket \sigma_1' \rrbracket <: \llbracket \sigma_1 \rrbracket$ again by definition of subtyping, so by induction, we can conclude that $v' \in \llbracket \sigma_2 \rrbracket$ as desired.

- Case (iter1): If the derivation is of the form

$$\frac{}{\Gamma \vdash_{\text{iter}} \{ () \} s \{ () \}} \quad \frac{}{\gamma; () \vdash \text{iter}[s] \Rightarrow^U ()}$$

then the conclusion is immediate.

- Case (iter2): If the derivation is of the form

$$\frac{\Gamma \vdash^1 \{ \alpha \} s \{ \tau \}}{\Gamma \vdash_{\text{iter}} \{ \alpha \} s \{ \tau \}}$$

then by assumption, $v \in \llbracket \alpha \rrbracket$. Hence, $v = t, ()$, so by induction hypothesis (2), we have $\gamma; t \vdash s \Rightarrow^U v'$ with $v' \in \llbracket \tau \rrbracket$ and can derive

$$\frac{\gamma; t \vdash s \Rightarrow^U v' \quad \frac{}{\gamma; () \vdash \text{iter}[s] \Rightarrow^U ()}}{\gamma; t, () \vdash \text{iter}[s] \Rightarrow^U v', ()}$$

- Case (iter3): If the derivation is of the form

$$\frac{\Gamma \vdash_{\text{iter}} \{ \tau_1 \} s \{ \tau_1' \} \quad \Gamma \vdash_{\text{iter}} \{ \tau_2 \} s \{ \tau_2' \}}{\Gamma \vdash_{\text{iter}} \{ \tau_1, \tau_2 \} s \{ \tau_1', \tau_2' \}}$$

then we must have $v = v_1, v_2$ where $v_i \in \llbracket \tau_i \rrbracket$ for $i \in \{1, 2\}$; by induction we have $\gamma; v_i \vdash \text{iter}[s] \Rightarrow^U v'_i$, where $v'_i \in \llbracket \tau_i' \rrbracket$ for $i \in \{1, 2\}$. Hence, by Lemma 3 we can conclude $\gamma; v_1, v_2 \vdash \text{iter}[s] \Rightarrow^U v'_1, v'_2$ where $v'_1, v'_2 \in \llbracket \tau_1', \tau_2' \rrbracket$.

- Case (iter4): If the derivation is of the form

$$\frac{\Gamma \vdash_{\text{iter}} \{ \tau_1 \} s \{ \tau_1' \} \quad \Gamma \vdash_{\text{iter}} \{ \tau_2 \} s \{ \tau_2' \}}{\Gamma \vdash_{\text{iter}} \{ \tau_1 | \tau_2 \} s \{ \tau_1' | \tau_2' \}}$$

then we must have $v \in \llbracket \tau_1 \rrbracket \cup \llbracket \tau_2 \rrbracket$; the cases are symmetric, so without loss suppose $v \in \llbracket \tau_1 \rrbracket$. By induction we have that $\gamma; v \vdash \text{iter}[s] \Rightarrow^U v'$ where $v' \in \llbracket \tau_1' \rrbracket <: \llbracket \tau_1 | \tau_2 \rrbracket$.

- Case (iter5): If the derivation is of the form

$$\frac{\Gamma \vdash_{\text{iter}} \{ \tau_1 \} s \{ \tau_2 \}}{\Gamma \vdash_{\text{iter}} \{ \tau_1^* \} s \{ \tau_2^* \}}$$

then since $v \in \tau^*$ we must have that either $v = ()$ (in which case the conclusion is immediate) or $v = v_1, \dots, v_n$ where each $v_i \in \llbracket \tau \rrbracket$. By induction, we can obtain derivations $\gamma; v_i \vdash \text{iter}[s] \Rightarrow^U v'_i$ where $v'_i \in \llbracket \tau_2 \rrbracket$ for each $i \in \{1, \dots, n\}$; hence, by repeated application of Lemma 3 we can conclude that $\gamma; v \vdash \text{iter}[s] \Rightarrow^U v'$, where by definition $v' = v'_1, \dots, v'_n \in \llbracket \tau_2^* \rrbracket$.

- Case (iter6): If the derivation is of the form

$$\frac{\Gamma \vdash_{\text{iter}} \{ E(X) \} s \{ \tau \}}{\Gamma \vdash_{\text{iter}} \{ X \} s \{ \tau \}}$$

then we have that $v \in \llbracket X \rrbracket = \llbracket E(X) \rrbracket$ so the induction hypothesis applies directly and we can conclude that $v' \in \llbracket \tau \rrbracket$.

This exhausts all cases and completes the proof. \square

C. Normalizing and typechecking source updates

Figures 14, 15, 16, and 17 show the main typechecking judgments for source statements, simple updates, and paths. The statement typechecking rules are straightforward; note however that we require that both statements and updates start and end with atomic types. The simple update typechecking rules each follow the procedure outlined above. The path typechecking rules match p against the input type α . Note that paths may bind variables, and the same variable may be bound to different types for different cases; this is why we need to include contexts Γ in the substitutions Θ . In certain rules, we choose fresh type variables. The scope with respect to which we require freshness is all type variables mentioned elsewhere in the surrounding derivation.

For many of the typechecking judgments, we need to typecheck an expression against all of the bindings of a context-tagged substitution Θ . We therefore introduce several *simultaneous typechecking* judgments shown in Figure 18.

C.1 Metatheory

For this source language, we first need some auxiliary lemmas to establish soundness.

Lemma 4. *If $\vdash^1 \{\Theta \oplus x\} s \{\Theta' \oplus x\}$ then $\vdash^1 \{\Theta\} \text{ snapshot } x \text{ in } s \{\Theta'\}$.*

Proof. Induction on derivation of $\vdash^1 \{\Theta \oplus x\} s \{\Theta' \oplus x\}$. \square

We also need an auxiliary notation $\Theta|\Theta'$, which merges two context-tagged type substitutions provided their bindings and contexts match:

$$(\Theta, Z \mapsto (\Gamma \triangleright \tau)) | (\Theta', Z \mapsto (\Gamma \triangleright \tau')) = \emptyset | \emptyset = \emptyset \\ (\Theta, Z \mapsto (\Gamma \triangleright \tau)) | (\Theta', Z \mapsto (\Gamma \triangleright \tau')) = (\Theta|\Theta'), Z \mapsto (\Gamma, \tau|\tau')$$

Lemma 5. *If $\vdash \{\Theta\} e : \text{bool}$ and $\vdash^1 \{\Theta\} s \{\Theta'\}$ then $\vdash^1 \{\Theta\} \text{ if } e \text{ then } s \text{ else skip } \{\Theta|\Theta'\}$.*

Proof. Induction on derivation of $\vdash^1 \{\Theta\} s \{\Theta'\}$. \square

Lemma 6. 1. *If the free type variables of τ are disjoint from those of Θ' then $\tau(\Theta \uplus \Theta') = \tau(\Theta)$.*
2. *If $\Gamma \vdash \tau :: \phi \rightsquigarrow (\tau', \Theta)$ then $\tau = \tau'(\Theta)$.*
3. *If $\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta)$ then $\alpha = \alpha'(\Theta)$.*
4. *If $\vdash_{Path} \{\Theta\} p \rightsquigarrow (\Theta', \Theta'')$ then $\Theta = \Theta'(\Theta'')$.*

Proof. For part (1), proof is by induction on the structure of types. For part (2), proof is by induction on the structure of derivations, using part (1) for the cases involving types τ_1, τ_2 and $\tau_1|\tau_2$. Parts (3) and (4) follow by simultaneous induction on derivations. \square

Lemma 7. *If $\Gamma \vdash \tau :: \phi \rightsquigarrow (\tau', \Theta)$ and $\vdash^1 \{\Theta\} s \{\Theta'\}$ then $\Gamma \vdash_{iter} \{\tau\} \phi?s \{\tau'(\Theta')\}$.*

Proof. Induction on derivation of $\Gamma \vdash \tau :: \phi \rightsquigarrow (\tau', \Theta)$. \square

Theorem 9 (Soundness). *Assume $\Gamma, \alpha, \alpha', \Theta_1$ have no free type variables Z .*

1. *If $\Gamma \vdash_{Stmnt} \{\tau\} s \{\tau'\}$ then $\Gamma \vdash^1 \{\tau\} [s]_{Stmnt} \{\tau'\}$.*
2. *If $\Gamma \vdash_{Upd} \{\alpha\} u \{\alpha'\}$ then $\Gamma \vdash^1 \{\alpha\} [u]_{Upd} \{\alpha'\}$.*
3. *If $\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta)$ and $\vdash^1 \{\Theta\} s \{\Theta'\}$ then $\Gamma \vdash^1 \{\alpha\} [p]_{Path}(s) \{\alpha'(\Theta')\}$.*
4. *If $\vdash_{Path} \{\Theta_1\} p \rightsquigarrow (\Theta_2, \Theta_3)$ and $\vdash^1 \{\Theta_3\} s \{\Theta'_3\}$ then $\vdash^1 \{\Theta_1\} [p]_{Path}(s) \{\Theta_2(\Theta'_3)\}$.*

Now, to prove completeness, we need lemmas establishing that the earlier lemmas are invertible:

Lemma 8. *If $\vdash^1 \{\Theta\} \text{ snapshot } x \text{ in } s \{\Theta'\}$ then $\vdash^1 \{\Theta \oplus x\} s \{\Theta' \oplus x\}$.*

$$\boxed{\Gamma \vdash_{Stmnt} \{\tau\} s \{\tau'\}} \\ \frac{\Gamma \vdash_{Stmnt} \{\tau\} s_1 \{\tau'\} \quad \Gamma \vdash_{Stmnt} \{\tau'\} s_2 \{\tau''\}}{\Gamma \vdash_{Stmnt} \{\tau\} s_1; s_2 \{\tau''\}} \\ \frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash_{Stmnt} \{\alpha\} s \{\tau'\}}{\Gamma \vdash_{Stmnt} \{\tau\} \text{ IF } e \text{ THEN } s \{\tau|\tau'\}} \\ \frac{\Gamma \vdash e : \tau_0 \quad \Gamma, x : \tau +_0 \vdash_{Stmnt} \{\tau\} s \{\tau'\}}{\Gamma \vdash_{Stmnt} \{\tau\} \text{ LET } x = e \text{ IN } s \{\tau'\}} \\ \frac{\Gamma \vdash_{Upd} \{\alpha\} u \{\alpha'\}}{\Gamma \vdash_{Stmnt} \{\alpha\} u \{\alpha'\}}$$

Figure 14. Typechecking rules for compound updates

Proof. Induction on the structure of derivations of

$$\vdash^1 \{\Theta\} \text{ snapshot } x \text{ in } s \{\Theta'\}$$

followed by inversion. \square

Lemma 9. *If $\vdash^1 \{\Theta\} \text{ if } e \text{ then } s \text{ else skip } \{\Theta'\}$ then there exists Θ'' such that $\Theta' = \Theta|\Theta''$ and $\vdash \{\Theta\} e : \text{bool}$ and $\vdash^1 \{\Theta\} s \{\Theta''\}$.*

Proof. Induction on the structure of derivations of

$$\vdash^1 \{\Theta\} \text{ if } e \text{ then } s \text{ else skip } \{\Theta'\}$$

followed by inversion. \square

Lemma 10. *If $\Gamma \vdash_{iter} \{\tau\} \phi?s \{\tau'\}$ then there exists τ'', Θ, Θ' such that $\tau''(\Theta') = \tau', \Gamma \vdash \tau :: \phi \rightsquigarrow (\tau'', \Theta)$ and $\vdash^1 \{\Theta\} s \{\Theta'\}$.*

Proof. Induction on the structure of derivations of

$$\Gamma \vdash_{iter} \{\tau\} \phi?s \{\tau'\}$$

and then using inversion and properties of substitutions. \square

Theorem 10 (Completeness).

1. *If $\Gamma \vdash^1 \{\tau\} [s]_{Stmnt} \{\tau'\}$ then $\Gamma \vdash_{Stmnt} \{\tau\} s \{\tau'\}$.*
2. *If $\Gamma \vdash^1 \{\alpha\} [u]_{Upd} \{\alpha'\}$ then $\Gamma \vdash_{Upd} \{\alpha\} u \{\alpha'\}$.*
3. *If $\Gamma \vdash^1 \{\alpha\} [p]_{Path}(s) \{\alpha'\}$ then there exist $\alpha'', \Theta, \Theta'$ such that $\alpha' = \alpha''(\Theta), \Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta)$, and $\vdash^1 \{\Theta\} s \{\Theta'\}$.*
4. *If $\vdash^1 \{\Theta\} [p]_{Path}(s) \{\Theta'\}$ then there exists $\Theta_1, \Theta_2, \Theta'_2$ such that $\Theta' = \Theta_1(\Theta'_2), \vdash_{Path} \{\Theta\} p \rightsquigarrow (\Theta_1, \Theta_2)$, and $\vdash^1 \{\Theta_2\} s \{\Theta'_2\}$.*

Proof. Parts (3) and (4) follow by simultaneous induction using previous lemmas. Parts (1) and (2) then follow by simultaneous induction, using parts (3) and (4). \square

$\Gamma \vdash_{Upd} \{\alpha\} u \{\alpha'\}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{left}[\text{insert } e] \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{INSERT BEFORE } p \text{ VALUE } e \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{right}[\text{insert } e] \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{INSERT AFTER } p \text{ VALUE } e \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{children}[\text{left}[\text{insert } e]] \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{INSERT AS FIRST INTO } p \text{ VALUE } e \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{children}[\text{right}[\text{insert } e]] \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{INSERT AS LAST INTO } p \text{ VALUE } e \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{delete} \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{DELETE } p \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{children}[\text{delete}] \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{DELETE FROM } p \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{rename } n \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{RENAME } p \text{ TO } n \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{delete; insert } e \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{REPLACE } p \text{ WITH } e \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash^1 \{\Theta\} \text{children}[\text{delete; insert } e] \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{REPLACE IN } p \text{ WITH } e \{\alpha' \langle \Theta' \rangle\}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash_{Stmnt} \{\Theta\} s \{\Theta'\}}{\Gamma \vdash_{Upd} \{\alpha\} \text{UPDATE } p \text{ BY } s \{\alpha' \langle \Theta' \rangle\}}$

Figure 15. Typechecking rules for simple updates

$\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta)$
$\frac{\Gamma \vdash_{Path} \{\alpha\} . \rightsquigarrow (\alpha, \emptyset)}{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta_1) \quad \vdash_{Path} \{\Theta_1\} p' \rightsquigarrow (\Theta_2, \Theta'_2)}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p/p' \rightsquigarrow (\alpha' \langle \Theta_2 \rangle, \Theta'_2)}{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta) \quad \vdash \{\Theta\} e : \text{bool}}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} p[e] \rightsquigarrow (\alpha' \langle \Theta'' \rangle, \Theta)}{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta)}$
$\frac{\Gamma \vdash_{Path} \{\alpha\} x \text{ as } p \rightsquigarrow (\alpha', \Theta \oplus x)}{\Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta)}$
$\frac{\Gamma \vdash_{Path} \{n[\tau]\} \phi \rightsquigarrow (n[\tau'], \Theta)}{\Gamma \vdash_{Path} \{n[\tau]\} \phi \rightsquigarrow (n[\tau'], \Theta)}$

Figure 16. Typechecking rules for paths

$\Gamma \vdash \tau :: \phi \rightsquigarrow (\tau', \Theta)$
$\frac{\Gamma \vdash () :: \phi \rightsquigarrow ((), \emptyset)}{\Gamma \vdash \alpha :: \phi \rightsquigarrow (\alpha, \emptyset)}$
$\frac{\alpha <: \phi \quad Z \text{ fresh}}{\Gamma \vdash \alpha :: \phi \rightsquigarrow (Z, Z \mapsto (\Gamma \triangleright \alpha))}$
$\frac{\Gamma \vdash \tau_1 :: \phi \rightsquigarrow (\tau'_1, \Theta_1) \quad \Gamma \vdash \tau_2 :: \phi \rightsquigarrow (\tau'_2, \Theta_2)}{\Gamma \vdash \tau_1, \tau_2 :: \phi \rightsquigarrow ((\tau'_1, \tau'_2), \Theta_1 \uplus \Theta_2)}$
$\frac{\Gamma \vdash \tau_1 :: \phi \rightsquigarrow (\tau'_1, \Theta_1) \quad \Gamma \vdash \tau_2 :: \phi \rightsquigarrow (\tau'_2, \Theta_2)}{\Gamma \vdash \tau_1 \tau_2 :: \phi \rightsquigarrow (\tau'_1 \tau'_2, \Theta_1 \uplus \Theta_2)}$
$\frac{\Gamma \vdash \tau_1 :: \phi \rightsquigarrow (\tau_2, \Theta)}{\Gamma \vdash \tau_1^* :: \phi \rightsquigarrow (\tau_2^*, \Theta)}$
$\frac{\Gamma \vdash E(X) :: \phi \rightsquigarrow (\tau', \Theta)}{\Gamma \vdash X :: \phi \rightsquigarrow (\tau', \Theta)}$

Figure 17. Typechecking rules for path filters

$\vdash \{\Theta\} e : \tau$
$\frac{\vdash \{\emptyset\} e : \tau \quad \vdash \{\Theta\} e : \tau \quad \Gamma \vdash e : \tau}{\vdash \{\emptyset, Z \mapsto (\Gamma \triangleright \alpha)\} e : \tau}$
$\vdash^1 \{\Theta\} s \{\Theta'\}$
$\frac{\vdash^1 \{\emptyset\} s \{\emptyset\} \quad \vdash^1 \{\Theta\} s \{\Theta'\} \quad \Gamma \vdash^1 \{\alpha\} s \{\tau\}}{\vdash^1 \{\emptyset, Z \mapsto (\Gamma \triangleright \alpha)\} s \{\Theta', Z \mapsto (\Gamma \triangleright \tau)\}}$
$\vdash_{Stmnt} \{\Theta\} s \{\Theta'\}$
$\frac{\vdash_{Stmnt} \{\emptyset\} s \{\emptyset\} \quad \vdash_{Stmnt} \{\Theta\} s \{\Theta'\} \quad \Gamma \vdash_{Stmnt} \{\tau\} s \{\tau'\}}{\vdash_{Stmnt} \{\emptyset, Z \mapsto (\Gamma \triangleright \tau)\} s \{\Theta', Z \mapsto (\Gamma \triangleright \tau')\}}$
$\vdash_{Path} \{\Theta\} p \rightsquigarrow (\Theta', \Theta'')$
$\frac{\vdash_{Path} \{\emptyset\} p \rightsquigarrow (\emptyset, \emptyset)}{\vdash_{Path} \{\Theta\} p \rightsquigarrow (\Theta', \Theta'') \quad \Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta''')}$
$\frac{\vdash_{Path} \{\Theta\} p \rightsquigarrow (\Theta', \Theta'') \quad \Gamma \vdash_{Path} \{\alpha\} p \rightsquigarrow (\alpha', \Theta''')}{\vdash_{Path} \{\Theta, Z \mapsto (\Gamma \triangleright \alpha)\} p \rightsquigarrow (\Theta', Z \mapsto (\Gamma \triangleright \alpha'), \Theta'' \uplus \Theta''')}$

Figure 18. Simultaneous typechecking judgments